A Study on Three-Dimensional Progressive Failure Analysis of Concrete Using Lattice Model

S. Kokusho , A. Wada and H. Kubota.
Tokyo Institute of Technology
2-12-1, O-okayama, Megro-ku, Tokyo 152, Japan

SUMMARY

A new model for concrete analysis making use of a lattice model is in this study. Cracks is con sidered as the dissection caused by the tension of the lattice elements. The noncontinuity and nonuniformity of concrete is expressed by adding the effect of random digits to the tensile strengths of these lattice elements and to the layout of nodal points. Analyses of circular cylinders are made by way of examination, and are compared with the finite element analysis.

1. INTRODUCTION

For the purpose of clarifying, through a numerical analysis (progressive failure analysis), the behavior of a reinforced concrete structure being subjected to an extensive, external disturbance until the ultimate failure of the structure, it is necessary to use a concrete model which permits the explanation of a compression failure of concrete, the effect of reinforcing bars to constrain the concrete, and a phenomenon that hoops constrain the buckling of main bars.

Today, a number of concrete models have been developed, and stress analyses using these models have been extensively done. However, some of these models are not perfect for structural analysis, mainly because of incompleter material models. So far, there is no constitutive equation in existence which can thoroughly express the fundamental properties of reinforced concrete materials and which can also be considered as being general.

The main reason which makes for the induction of such a constitutive equation lies in that concrete is not uniform and

hence it is impossible to make a perfect prediction as to form what direction a crack develops. For this reason, these would be no possibility that such a constitutive relation could always be determined. It would rather be more natural to think the constitutive relation cannot be determined, but distributed with a certain probability.

The conceivable methods determining such a distribution can largely be classified into the following two;

(1) A method statistically determining the distribution taking into account the probability distribution when inducing an equation.

(2) A method called the Monte Carlo method which employs random digits for material constants, in lieu of using a simple model, so that value simulation can be made.

Comparing these two methods, while the former entails a difficulty in inducing a constitutive equation, the results obtained are considered much more general. However, the application of the former is at the present time limited to special cases, such as the state of plane stress, and difficult to be made for an actual structure in which reinforcing bars and concrete are arranged in complicated patterns. On the contrary, the Monte Carlo method is applicable to the structures of any configurations as far as a suitable model is envisaged.

The present study is proximate to the latter concept. First, it is considered that the failure of concrete results from locally expanded cracks and that a crack develops when the tensile force in a certain portion is in excess of a certain strength. By simplifying the failure of concrete in this way, concrete can be presumably replaced with a cubic lattice(or truss) subjected only to the axial force.[1,2,3] In this case, concrete cracks can be considered as the dissection caused by the tension of the lattice elements. The noncontinuity and nonuniformity of concrete can be expressed by adding the effect of random digits to the tensile strengths of these lattice elements and to the layout of nodal points.

2. LATTICE MODEL ANALYSIS

As previously described in the preceding paragraph, the present study represents concrete by means of lattice model.

The arrangement of model points will be accomplished such that after nodal points have been arranged at the lattice defect points, the direction toward which the nodal points are to be moved is determined, and then the magnitude obtained by the random digits is moved to that direction. In this case, the spacing of the nodal points should be approximately from 0.5 cm to 2 cm with consideration to the size of the aggregates in the concrete.

The relationship between the axial force of concrete lattice elements, N, and the deformation in axial direction, will be of the bi-linear type as shown on Fig. 1. The dissection by means of tension will be performed by replacing the loading and unloading of the entire external forces and by reducing the axial forces substained by the elements while continuously providing the elements with positive stiffness. In this case, the compressive side is assumed to maintain linear elasticity.

From the experimental results that the uniaxial compressive strength of concrete begins developing at an approximate 0.2% strain in the axial direction and from the fact that a crack develops in parallel to the compression direction when uniaxial compression occurs, the present study will determine the strengths of elements to be as follows:

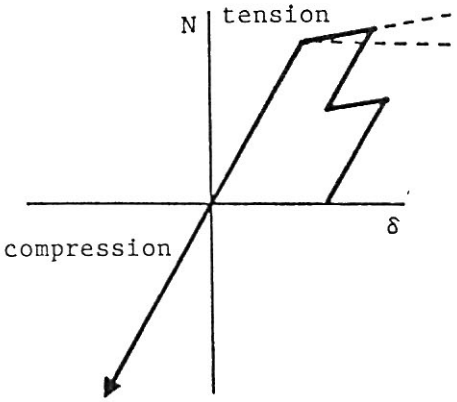


Fig.1 N-δ relationship of concrete element

A kind of Poisson's ratio is obtained from the ratios of vertical displacement and horizontal displacement which are derived from the elastic analysis of the truss structure shown in Fig. 2, and then the strains of the horizontal elements at the time of the vertical strain resulting from the preceding calculation being at 0.2% is assumed to be the average maximum strain of the elements. By this average maximum strain, the length of that element, and the sectional area derived from proper random digits and the modulus of elasticity common to all concrete elements, the strength of elements will be determined.

Fig. 3 shows a cubic which is made from lattice model, and Fig.4 shows the flow chart of this method.

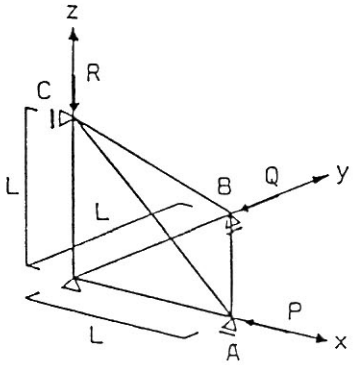


Fig.2 A basic structure of lattice model

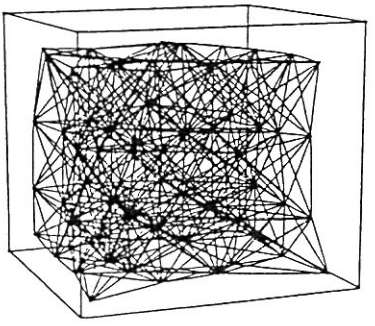
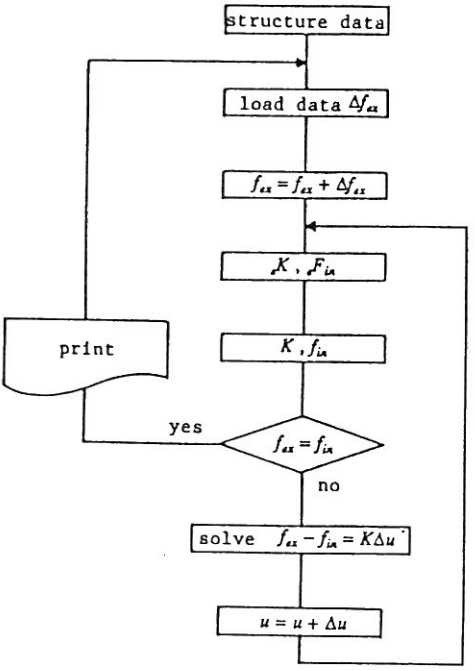
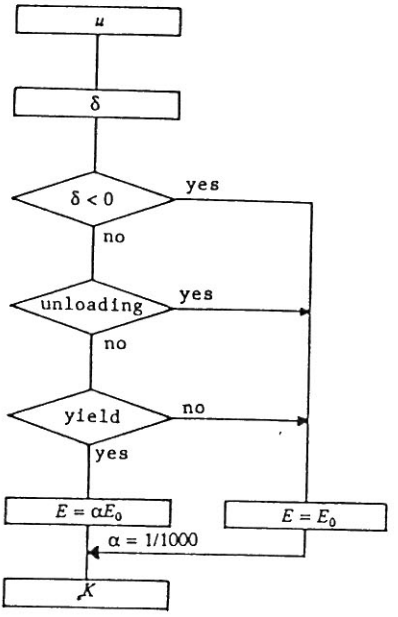


Fig.3 A cubic made from lattice model



(1) Overall



(2) Making rule of element stiffness matrix

Fig.4 Flow chart

3. ANALYSIS EXAMPLE USING A LATTICE MODEL

This example involves a circular cylinder having a diameter of 10 cm and a height of 20 cm, which is normally used for a compressive strength test of concrete. As shown in Fig. 5, the analysis covered one-fourth of the column. The analysis was made with the top and bottom ends of the column were constrained with respect to the displacement in horizontal direction, and was made such that the model was loaded vertically in a manner to control the displacement of the model.

Analysis was made of three cases (test pieces No. 1 through No. 3) for observing the effects of variations. While test pieces No. 1 and No. 2 are arranged at the same nodal points, differing in the strength of elements, test piece No. 3 differs in the variation of both nodal point arrangement and element strength. Fig. 6 shows the relationship between the average stress in the axial direction, , and the average stress in the axial direction, . Fig. 7 shows the behavior of test piece No. 1 where its element is being dissected as its loading is increasing.

The results indicate that from the softening of the concrete stiffness to the stage of so-called negative modulus can be represented only by considering the dissection of the elements on the tension side while taking account of the compressive failure of the concrete elements. In addition,

from the fact that despite models No. 1 ,No. 2 and No. 3 being completely different, they all display nearly the same tendency, the application of this analysis method would enable the representing of some tendency without the analysis of many cases.

Of the three cases, the Poisson's ratio of test piece No. 2 was obtained by the following method.

A layer of approximately 4 cm thick in the area at the center of the test piece which is seldom affected by the constrained top and bottom is envisaged, and then a horizontal strain is obtained from the displacement, toward the radius direction, of the nodal points located at the perimeter included in the layer (Fig. 8).

Fig. 9 shows the relationship between Poisson's ratio and the average stress in the axial direction. The value obtained by the present analysis is greater than the value 1/6, and proximate to the value (1/5) established by DIN or the factor of about 0.2 obtained by Chen's experimental result[4] (Fig. 10). As such, despite that the analysis does not directly introduce Poisson's ratio, the value obtained by the present analysis gives most expression as a result, including the phenomenon that Poisson's ratio also becomes larger as it gets closer to the maximum strength.

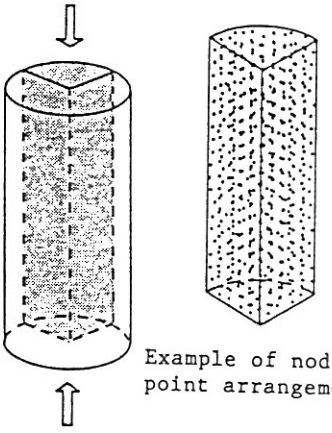


Fig.5 Concrete cylinder

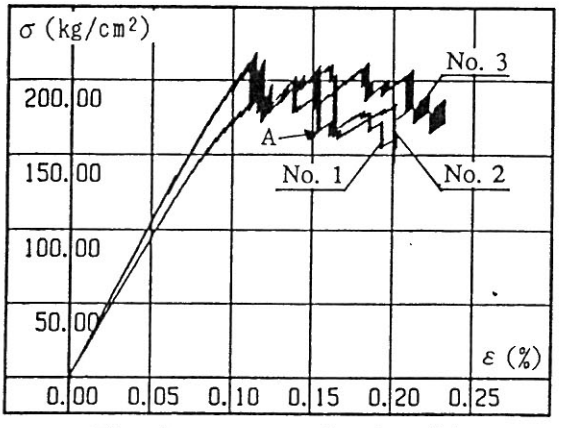


Fig.6 σ－ε relationship of concrete cylinder

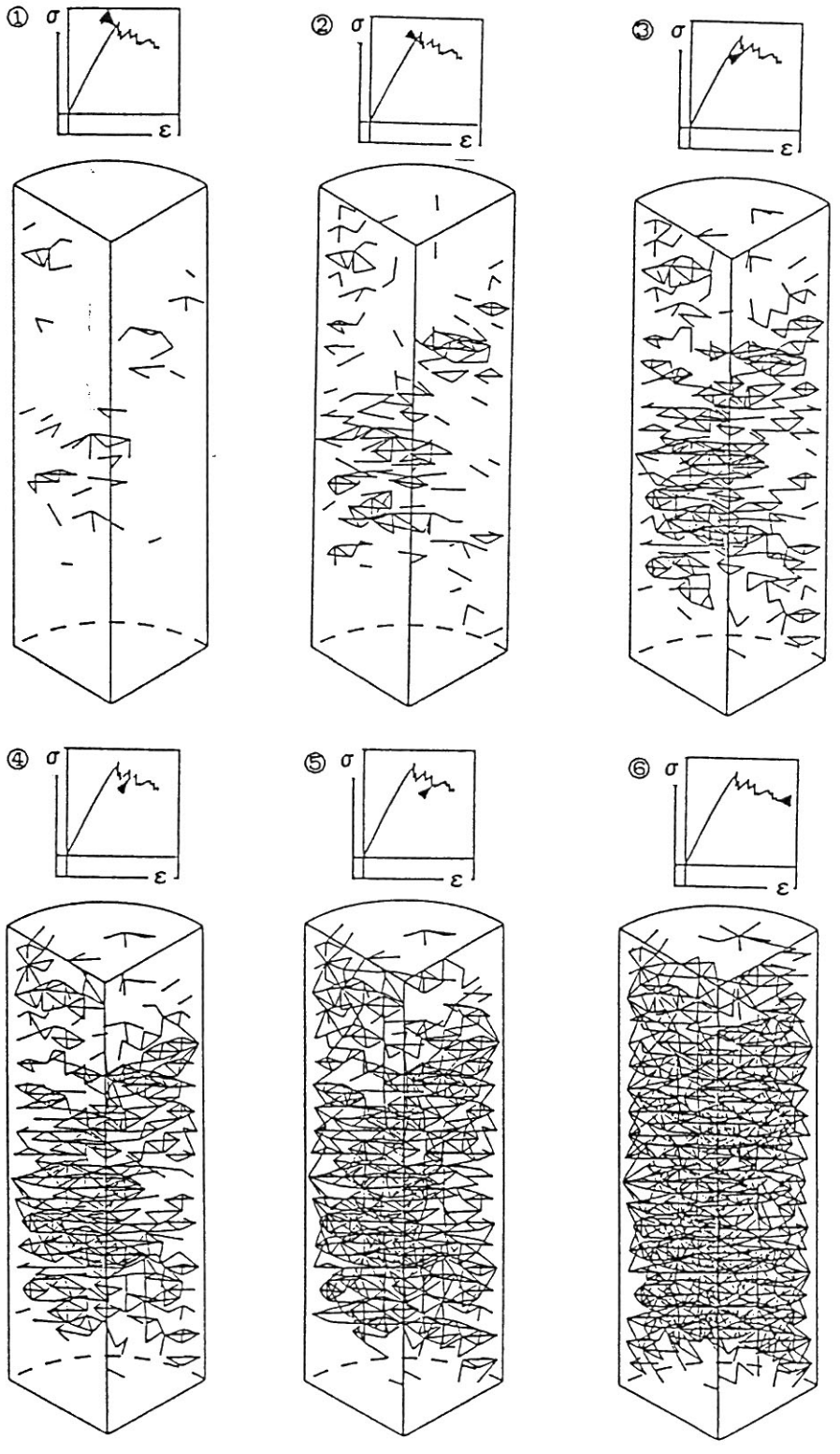


Fig.7 Dissected elements until each steps

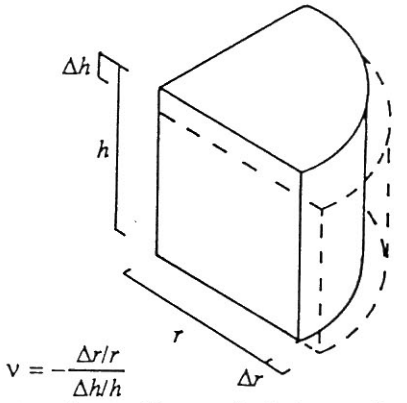


Fig.8 Define of Poisson's ratio

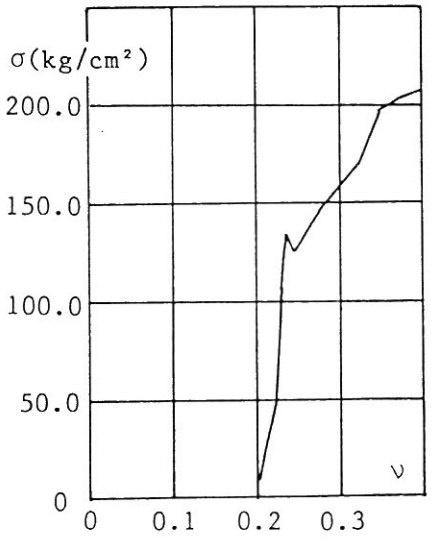


Fig.9 Relationship between Poisson's ratio and average stress

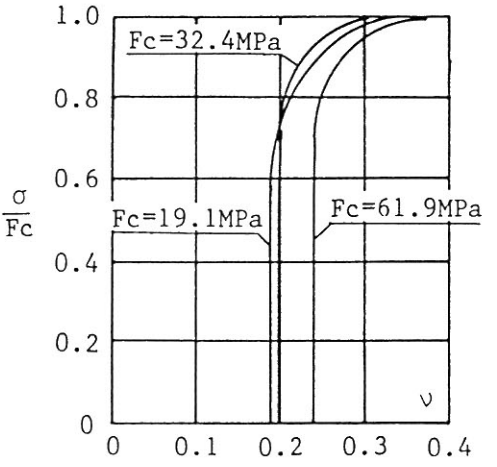


Fig.10 Experimental results of relationship between Poisson's ration and stress (Chen[5])

4. COMPARISON WITH FINITE ELEMENT ANALYSIS

4.1 Finite element model covered by analysis

The analysis results obtained through the abovementioned model were compared with the results by finite element method. The element used was a three-dimensional isoparametric solid element, which was made with reference to the Bathe's [5] model, but was arranged in failure condition by us. The outline will be as described below.

(a) The relation ship between the uniaxial stress and the stress

The relationship between the uniaxial stress σ and the stress ε will be as shown in the figure, namely,

(i) In $\varepsilon \leq 0$ (compression zone), σ will be expressed as a parameter of ε as

$$\sigma/\sigma_c=\frac{(E_0/E_s)(\varepsilon/\varepsilon_c)}{1+A(\varepsilon/\varepsilon_c)+B(\varepsilon/\varepsilon_c)^2+C(\varepsilon/\varepsilon_c)^3} \tag{1}$$

In addition, stiffness E is the derivative of this function. Here,

$$A=\frac{E_0/E_u+(p^3-2p^2)E_0/E_s-(2p^3-3p^2+1)}{(p^2-2p+1)p} \tag{2}$$

$$B=2E_0/E_s-3-2A \tag{3}$$

$$C=2-E_0/E_s+A \tag{4}$$

E_0: initial stiffness
σ_c: maximum compressive stress
ε_c: strain at the maximum compressive stress
σ_u: stress at compression failure
ε_u: strain at compression failure

$$E_s=\frac{\sigma_c}{\varepsilon_c} \tag{5}$$

$$p=\varepsilon_u/\varepsilon_c \tag{6}$$

$$E_u=\sigma_u/\varepsilon_u \tag{7}$$

Provided, however, that the initial stiffness is used when when the load is relieved.

(ii) In $\varepsilon > 0$ (tension zone), the stress-strain relationship is linear and its stiffness is constant with the initial stiffness E_0 .

(iii) Poisson's ratio is always constant.

(b) Element stiffness matrix under the state of multiaxial stress

Under the state of multiaxial stress, the stiffness matrix of element will be established in the following order:.

(i) Principle stress σ_{pi} and its direction will be sought. $(\sigma_{p3} \leq \sigma_{p2} \leq \sigma_{p1})$

(ii) The parameter γ for the stress representing the failure envelope will be established as

$$\gamma=a(\sigma_{p2}/\sigma_c)^2+b(\sigma_{p2}/\sigma_c)+c \tag{8}$$

a, b and c will be established in a manner that γ pass through the following three points:

$(1.00(\sigma_{p1}/\sigma_c), 1.00+1.50(\sigma_{p1}/\sigma_c))$

$(0.75(\sigma_{p1}/\sigma_c), 1.25+1.70(\sigma_{p1}/\sigma_c))$

$(1.00(\sigma_{p1}/\sigma_c), 1.20+1.60(\sigma_{p1}/\sigma_c))$

Assuming that the value obtained by multiplying σ_c, σ_u, ε_c and ε by γ are the material constants at the time, respectively, the stiffness will be established from the derivative in equation (1).

(iii) Orthotropical stress-strain matrix will be assembled from the stiffness in the direction of principle stress.

4.2 Analysis results using finite element method

The analysis using the analysis model described in paragraph 4.1 covered one-fourth of the same circular column as that described in paragraph 3. Fig. 11 shows elements idealization. Analysis was made such that the model was loaded vertically in a manner to control the displacement of the model. Fig. 12 shows the relationship between the displacement(strain) in the axial direction and the load (stress). In the figure, the solid line represents the top and bottom ends being constrained not to cause horizontal displacement, and the broken line represents them without constraint. That with the top and bottom ends being constrained not to cause horizontal displacement yielded to failure with a load lower than that with them not being constrained. Fig. 13 shows the relationship between Poisson's ratio and the average stress in the axial direction.

5. COMPARISON IN ANALYSIS BETWEEN LATTICE MODEL ANALYSIS AND FINITE ELEMENT METHOD

5.1 Load(stress)-displacement(strain) relationship

The results of the analysis using the lattice model indicate a high load constantly because of the method of stress release (to keep positive stiffness after yielding, and to use the initial stiffness when being reverse loaded). The load (stress)-displacement(strain) relationship rather lowers by the repetition of loading and unloading, and the points (like point A in the Fig. 6) to which the relationship re-ascends are equal to the time when the reliese of stress has ended, so it would be considered that the line connecting this point (the bottom line of the envelopes), is equal to the result obtained by the present analysis.

Considering this way, it may be said that the analysis result until reaching peak point obtained by the use of the lattice model is proximate to the result obtained by the finite

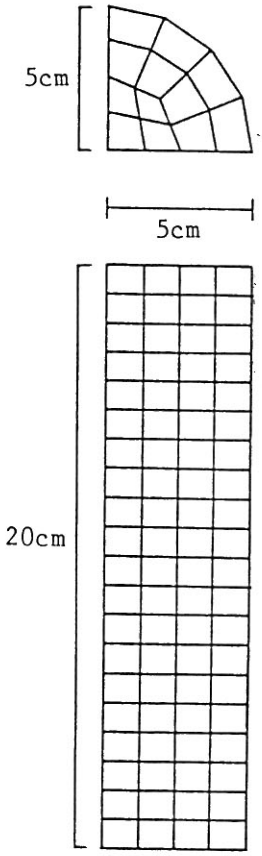


Fig.11 Finite element idealization

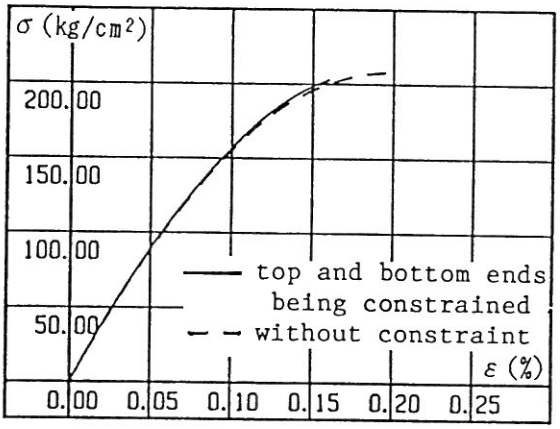


Fig.12 Load displacement curve of concrete cylinder (by FEM)

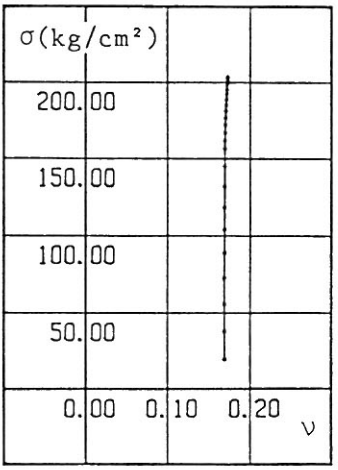


Fig.13 Relationship between Poisson's ratio and average stress(by FEM)

element method. After reaching peak point, on the other hand, it is difficult to analyse by the finite element method, but it is able to analyse by using lattice model.

5.2 Poisson's ratio

In the case of concrete, unlike metallic materials, a uniform Poison's ratio cannot be established because concrete consists of mixed ingredients and has a number of voids. In the case of lattice model, the analysis input contains no Poisson's ratio, but as a result of analysis Poisson's ratio was established from the ratio of vertical strain to

horizontal strain. The value obtained is proximate to the experimental result at the initial stage, and well resembles the experimental development in that as stress increases, the value also increases.

On the other hand, in the finite element analysis method, Poisson's ratio has to be given from the outside. In the present study, Poisson's ratio was assumed to be always 1/6 regardless of the stress condition. Although it is possible to change Poisson's ratio according to the stress condition, it is, one way or the other, greatly affected by the experimental result. In this regard, the initial stiffness alone relies on an experiment using a lattice model.

5.3 CPU time of calculation

As the computer which was used for the calculation is different from the one used for the other analysis, the ratio between exact time durations is unknown, but the time duration required for calculation by the finite element method is shorter by some 80%. However, this is by all means the result obtained by means of the current computer, and it is unknown that depending on a different computer architecture, which can be made faster, in the case of being simple and many elements or in the case of being a small number of complicated elements. (When more advenced parallel processing will be developed in the future, the calculation by means of a lattice model may probably be made faster.)

5.4 Variations

The lattice model is provided with variations in the stiffness of elements and in the arrangement of nodal points, representing the characteristic of concrete that the strength distribution and stiffness vary within the elements. However, the extent of the variations (standard deviation) could not be grasped, and therefore had to be properly determined. Reversely, analysis results can be used for the inference of the variation in concrete. On the other hand, the introducing of a variation to the finite element method is possible by changing the characters (stiffness and strength) at each gauss integral point, but a mathematical question would be left.

6. CONCLUSION

The foregoing has described the principles and their analysis examples of the progressive failure analysis of concrete using the lattice model. Also, comparison between the results obtained by the progressive analysis and those obtained by the finite element method was made, describing their characteristics, effectivenesses and futuristic possibilities. In the present analysis, the uniaxial compression alone was covered, and nothing is yet known of the behavior under the general stress condition. In the

future, this question, and the effectiveness under the condition of bending moment, shear stress or combined stress by means of this model will be studied so as to find out what constitutive relation will be derived from the result as well as how such a result can be used directly for structural analysis.

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