Computer Analysis of Inelastic Behavior of Reinforced Concrete Frame

Analyse par ordinateur du comportement inélastique d'un cadre en béton armé

Computeranalyse des inelastischen Verhaltens von Stahlbetonrahmen

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SUMMARY

This paper presents a method for considering the shear deformation of a beamcolumn joint and the continuity of the reinforcing bars through beams, columns, and beam-column joints. The authors have performed the analyses of a single-storied single-span reinforced concrete structure and a cross-shaped reinforced concrete structure. Analysis results compare favorably with experimental data for these structures.

RÉSUMÉ

L'étude décrit une méthode pour étudier la déformation due au cisaillement d'un joint de poutre-poteau et la continuité des armatures passant par les poutres et poteaux ainsi que les joints poutre-poteau. Les auteurs ont analysé une structure en béton armé à un étage et une portée et une structure en béton armé croisée. Les résultats des analyses correspondent aux données expérimentales.

ZUSAMMENFASSUNG

Die Studie befaßt sich mit einem Verfahren zur Berechnung der Verformung einer Balkenträgerverbindung und der Führung des Bewehrungsstahles durch Balken, Stützen und Rahmenknoten. Von den Autoren wurden Analysen an einem einstöckigen, einschiffigen Stahlbetonrahmen und an einem kreuzförmigen Stahlbetonknoten im Vergleich zu den experimentellen Daten mit guten Ergebnissen durchgeführt.

1. INTRODUCTION

Until now, most analysis methods which consider bond-slip or shear deformation of beam-column joints have been 2-dimensional or 3-dimensional methods, such as finite element method. Some researchers have made use of beam theory taking into consideration shear deformation of beam-column joints[1]. Others have taken into consideration bond-slip[2]-[4]. But there have been few analysis of reinforced concrete frame structures taking into consideration the continuity of reinforcing bars among beam, column and beam-column joints.

In this paper, we report on a technique using finite segment method, that considers the shear deformation of beam-column joints and the continuity of reinforcing bars among beam, column and beam-column joint.

2. ANALYTICAL METHOD

2.1 Modeling of Reinforced Concrete Frame and Analytical Assumptions

An outline of analytical model for a single beam-column joint is shown in Fig.1.

2.1.1 Reinforced concrete members

(a)The member is assumed to be of such type that traditional beam theory is applicable. But in order to approximate the curves of each member after deformation, the member is divided into 20 to 60 member elements.
(b)In each member element, only the bending deformation and the axial deformation is considered and shearing deformation is ignored.
(c)The distribution of stress intensity and strain in the cross section is considered by dividing the cross section into 34 layers as shown in Fig.2. The values of stress and strain in each unit element are represented by the values at the each center point.
(d)Stress and strains condition in member elements are traced solely at the sections at the ends of each member element[5].

2.1.2 Beam-column joints

Beam-column joints are treated as a joint panel. Only shear deformation is considered in the joint panel. In consideration of the bond-slip and yielding of the reinforcement in the joint panel, individual reinforcement in it will be divided into 20 elements in the axial direction of that reinforcement.

2.1.3 Mechanical properties of materials

The stress-strain relation of concrete and reinforcement is taken to follow, hysteresis loops as shown in Fig.3 and Fig.4 respectively. For the bond-slip relation, a hysteresis loop which is shown in Fig.5 is used. For the stress-

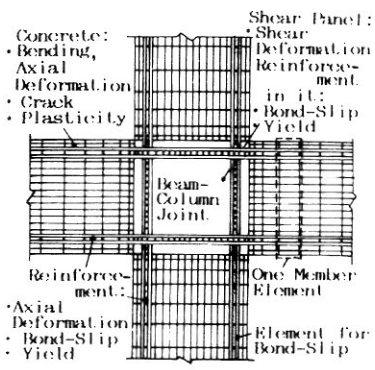


Fig.1 Reinforced concrete frame model

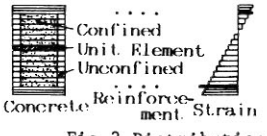


Fig.2 Distribution of strain and stress

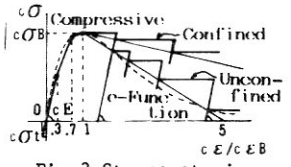


Fig.3 Stress-strain relationship of concrete

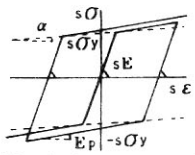


Fig.4 Stress-strain relationship of reinforcement

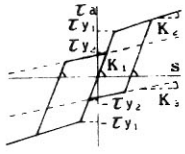


Fig.5 Bond-slip relationship

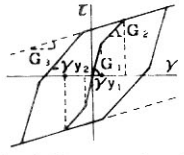


Fig.6 Stress-strain relationship of concrete panel

strain relation of the concrete panel, a hysteresis loop which is shown in Fig.6 is used.

2.1.4 Coordinates and incremental displacements

(a)Axes X and Y in the global coordinate system, and axes x and y in a local coordinate system associated with each of the member elements are defined. Incremental displacements Δu and Δv as the displacement constituents in the directions corresponding to axes x and y, incremental rotation $\Delta\theta$, and the incremental slip of individual reinforcement, Δs, will be considered at individual nodes (Fig.7).
(b)x_p and y_p with the origin positioned at the center of the joint panel are defined. Incremental displacements $\Delta_p u$ and $\Delta_p v$ as the displacement constituents in the directions corresponding to axes x_p and y_p and incremental rotations $\Delta_p\theta_b$ and $\Delta_p\theta_c$ of the beam and column surfaces will be considered at the center points of the joint panel. Regarding individual reinforcement, incremental slips $\Delta_p s$ will be considered at the 19 nodes inside, and the 2 nodes on the surfaces of the ends of the joint panel (Fig.8).

2.2 Analytical Method

The non-linear analytical method discussed in this paper is based on the incremental method, called initial stress method, which is derived from the stationary principle of incremental potential energy. As a result, the equation for each node is given in Eq.(1)

$$K\Delta u + f_{in} - f_{ex} = 0 \qquad \text{— (1)}$$

where K : The whole structure stiffness matrix
Δu : The incremental displacement in the global coordinate system of the whole structure
f_{in} : The internal force vector at the node
f_{ex} : The external force vector at the node

Δu is obtained by using Eq.(1). Then incremental strain in the structure induced by Δu is computed utilizing the local coordinate system at pre-incremental step and also, stress is modified according to such incremental strain. Thus, the computation procedures gain one step.

3. INCREMENTAL DISPLACEMENTS AND INCREMENTAL STRAINS

3.1 Incremental Displacement Inside a Member Element

The cross section which passes through points(x,0) on axis x will be considered. Incremental displacements in the directions corresponding to axes x and y at the points(x,0) on axis x will be taken as Δu and Δv, respectively and incremental rotation displacement will be taken as $\Delta\theta$. Incremental slip between the reinforcement and concrete in tier k within this cross section will be taken as Δs_k (Fig.7).
For concrete elements, the incremental displacements in the directions of x and y at arbitrary points x and y_k within the cross section, and incremental rotation displacements $\Delta_c u$, $\Delta_c v$ and $\Delta_c\theta$ can be expressed as shown in equations (2):

$$\begin{aligned} \Delta_c u &= \Delta u - y\frac{d\Delta u}{dx} \\ \Delta_c v &= \Delta v \\ \Delta_c\theta &= \Delta\theta \end{aligned} \qquad \text{— (2)}$$

For reinforcement x and y in tier k within this cross section, the incremental displacement in the direction of x and the incremental slips $\Delta_s u_k$ and Δs_k between the reinforcement and

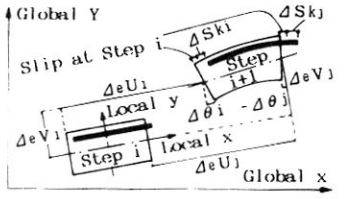


Fig.7 Deformation and coordinate of member element

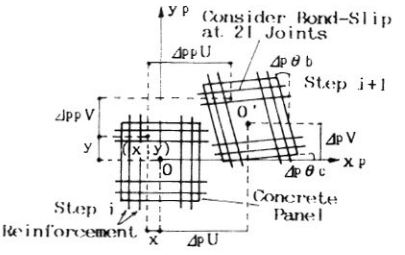


Fig.8 Deformation and coordinate of joint panel

concrete can be as given by eqs.(3) :

$$\begin{aligned} \Delta_s u_k &= \Delta u - y_k \frac{d\Delta u}{dx} + \Delta s_k \\ \Delta s_k &= \Delta s_k \end{aligned} \qquad \text{— (3)}$$

3.2 Incremental Shear Strain of A Concrete Panel

It is assumed that the concrete panel is subject to shear deformation only. Incremental shear strain $\Delta\gamma$ in the joint panel can be expressed as the difference between the incremental rotations of column and beam surfaces:

$$\Delta\gamma = \Delta_p\theta_c - \Delta_p\theta_b \qquad \text{— (4)}$$

where $\Delta_p\theta_c$: Incremental rotation of column surface
$\Delta_p\theta_b$: Incremental rotation of beam surface

3.3 Incremental Displacement of Reinforcement in Joint Panel

The incremental displacement $\Delta_{ps}u_b$, in the axial direction of x_p, of the beam reinforcement positioned between axes x_p and y within the joint panel, and the incremental displacement $\Delta_{ps}v_c$, in the axial direction of y_p, of the column reinforcement positioned between axes y_p and x can be expressed as given by equations (5) and (6):

[Beam Reinforcement] $\Delta_{ps}u_b = \Delta_p u - y_p\theta_b + \Delta_p s_b$ — (5)
[Column Reinforcement] $\Delta_{ps}u_c = \Delta_p v - x_p\theta_c + \Delta_p s_c$ — (6)

where $\Delta_p u$: Incremental displacement, in the axial direction of x_p, of the center point of the joint panel
$\Delta_p v$: Incremental displacement, in the axial direction of y_p, of the center point of the joint panel
$\Delta_p s_b$: Incremental slip of beam reinforcement
$\Delta_p s_c$: Incremental slip of column reinforcement

4. COMPATIBILITY OF INCREMENTAL DISPLACEMENTS BETWEEN MEMBER AND JOINT PANEL

The condition of compatibility of incremental displacement at center point P of the joint panel, $\Delta_p u=(\Delta_p u, \Delta_p v, \Delta_p\theta_b, \Delta_p\theta_c)^T$, and that at center point B on the beam surface, $\Delta u_b=(\Delta u_b, \Delta v_b, \Delta\theta_b)^T$, can be expressed as given by eq.(7) (Fig.9). :

$$\begin{bmatrix}\Delta u_b\\ \Delta v_b\\ \Delta\theta_b\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & \ell_x\\ 0 & 0 & 1 & 0\end{bmatrix} \begin{bmatrix}\Delta_p u\\ \Delta_p v\\ \Delta_p\theta_b\\ \Delta_p\theta_c\end{bmatrix} = R_b\,\Delta_p u \qquad \text{— (7)}$$

The condition of compatibility of incremental displacement at center point P of the joint panel, and that at center point C on the column surface, $\Delta u_c=(\Delta u_c, \Delta v_c, \Delta\theta_c)^T$, can be expressed as given by equation (8):

$$\begin{bmatrix}\Delta u_c\\ \Delta v_c\\ \Delta\theta_c\end{bmatrix} = \begin{bmatrix}1 & 0 & -\ell_y & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}\Delta_p u\\ \Delta_p v\\ \Delta_p\theta_b\\ \Delta_p\theta_c\end{bmatrix} = R_c\,\Delta_p u \qquad \text{— (8)}$$

where ℓ_x and ℓ_y will be given in the panel coordinate system (see Fig.9).

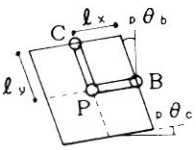


Fig.9 Joint panel

5. COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS

5.1 Single-storied Single-span Reinforced Concrete Structure

Matuzaki and his colleagues experimented with a single-storied single-span reinforced concrete structure. Figure 10 shows the specimen and Fig.11 shows idealized model for the analysis. Tensile force will be applied to the top of the left column when loading in the right direction, and to the top of the right column when loading in the left direction.

Table 1 shows the empirically established mechanical properties of materials and Fig.12 shows both analysis and experimental results for the relationship between load and the horizontal displacement at point ① (see Fig.11). The results of our analysis and agree very well with experimental values. Figure 13 shows the

diagram of deformation at point Ⓐ(P=34.4kN,δ=8.96mm) in Fig.12, the diagram of crack distribution, the diagram of reinforcement stress distribution, and the diagram of bond stress distribution.

Table 1 Mechanical properties

$_cE$ =24892.	MPa	$_sE$ =205800.	MPa
$_c\sigma_B$=27.2	MPa	E_b=2058.	MPa
$_c\varepsilon_B$=0.0023		α=1029.	MPa
K_1 =392.	MPa/cm	σ_y=394.9	MPa
K_2 =19.6	MPa/cm	G_1 =5880.	MPa
K_3 =4.9	MPa/cm	G_2 =4704.	MPa
τ_{y1}=3.92	MPa	G_3 =1176.	MPa
τ_{y2}=0.49	MPa	γ_{y1}=0.0005	
		γ_{y2}=0.003	

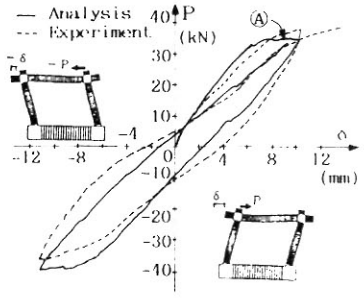


Fig.12 Relation of load-deflection

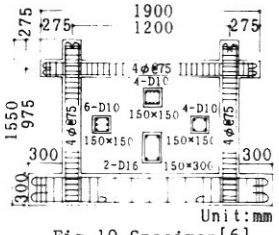


Fig.10 Specimen[6]

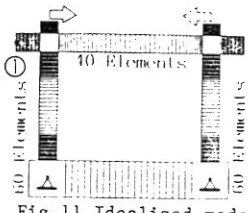
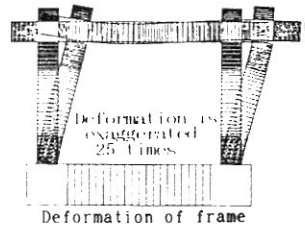
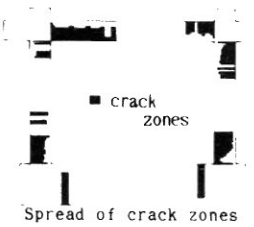


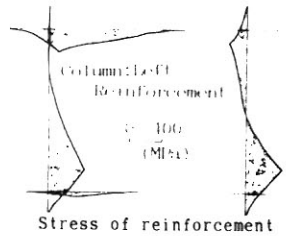
Fig.11 Idealized model for the analysis



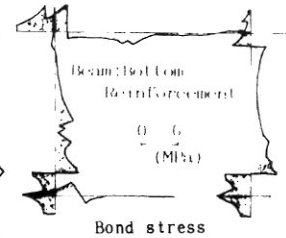
Deformation of frame



Spread of crack zones



Stress of reinforcement



Bond stress

Fig.13 Deformation, crack, reinforcement stress, and bond stress

5.2 Cross-shaped Reinforced Concrete Structure

In another experiment Shirouchi and his colleagues tested a cross-shaped reinforced concrete structure. Figure 14 shows the specimen and Fig.15 shows idealized model for the analysis. Table 2 shows the mechanical properties of materials. Fig.16 shows the relationship between the shearing force of the column and the horizontal displacement at the loading point, where the shearing force is that for which consideration is given to the effect of additional bending moment due to the axial force. Figure 17 shows the relationship between the shearing force of a column and the deflection of a beam derived from the rotation angle of that beam. Figure 18 shows the distribution diagram of beam reinforcement strain. Figure 19 shows the distribution diagram of cracks at point Ⓑ(P=179kN,δ=53.2mm) in Fig.16. The results of experiment and analysis again agree very closely.

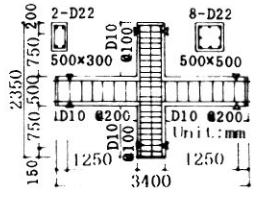


Fig.14 Specimen[7]

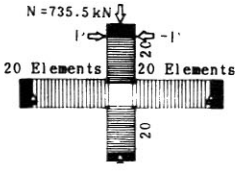


Fig.15 Idealized model for the analysis

Table 2 Mechanical properties

$_cE$ =25578.	MPa	$_sE$ =205800.	MPa
$_c\sigma_B$=28.6	MPa	E_b=2058.	MPa
$_c\varepsilon_B$=0.0027		α=1029.	MPa
K_1 =392.	MPa/cm	σ_y=376.3	MPa
K_2 =19.6	MPa/cm	G_1 =5880.	MPa
K_3 =4.9	MPa/cm	G_2 =4704.	MPa
τ_{y1}=3.92	MPa	G_3 =1176.	MPa
τ_{y2}=0.49	MPa	γ_{y1}=0.0005	
		γ_{y2}=0.003	

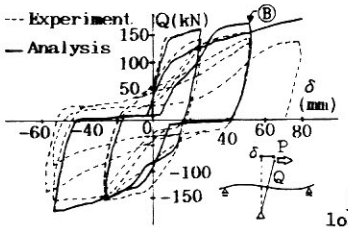


Fig.16 Relation of load-column deflection

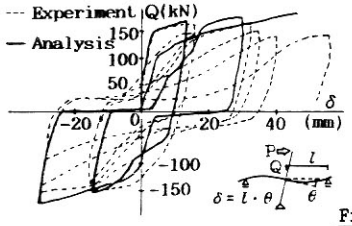


Fig.17 Relation of load-beam deflection

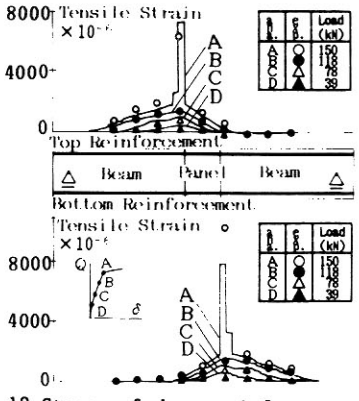


Fig.18 Stress of beam reinforcement

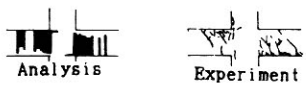


Fig.19 Spread of crack zones

6. CONCLUSIONS

1)By incorporating a joint panel, the shear deformation of beam-column joints can be considered while satisfying the continuity of the reinforcement through the beam-column joints.

2)Although only two analytical examples have been outlined, the results of experiment and analysis show a good correspondence with respect to the load-displacement relationship, reinforcement strain distribution and crack distribution. The analytical method dealt with in this paper would be effective for bending yield type reinforced concrete frames.

7. ACKNOWLEDGMENT

Dr. Yasuhiro Matuzaki, Dr. Shizuo Hayashi and Dr. Katsumi Kobayashi are acknowledged for their constant, useful guidances and provisions of valued experimental data. Messrs. Tetsuo Toda and Akinori Iwasaki, Hazama-Gumi, Ltd. are wholeheartedly acknowledged for permission to use their valued experimental data. Our study gratefully spent part of the grant in aid for scientific research from the Ministry of Education, Science and Culture of Japan, and those concerned are highly acknowledged.

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