Reprinted from Seismic Engineering: Research & Practice
Proceedings, Structures Congress '89, ASCE
San Francisco, CA/May 1-5, 1989

Building Frames Subjected To 2D Earthquake Motion

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Inelastic response of a simplified model of a 10-storey building frame was analyzed to assess the effects of bi-directional seismic loading. Interaction between the strengths in the two orthogonal directions was accounted for.

Introduction

In building frames designed to resist earthquakes, the girders usually have a lower yield moment than the columns. This arrangement distributes the inelastic action among many frame members, leading to good overall energy dissipation. By contrast, the damage in a weak-column structure is concentrated in one storey, with potentially disastrous results.

Studies on the column/girder strength ratio required for creating a weak-girder frame have been carried out by Akiyama[1], Kubo[2], and Teramoto[10]. They maintain that a column/girder strength ratio of 1.2 - 1.3 or more is required for the girder to yield.

These conclusions are based on analyses of structures subjected to one-directional seismic motion. However the behavior of frames subjected to bi-directional ground motion differs in three important respects. (i) larger column/girder strength ratios are required to prevent column yielding;[3,4,5,6,8,9,11] (ii) when moments are applied at 45° to the x-axis of the structure, the total strength of the four girder framing into each column is $2\sqrt{2}\,GM_y$, whereas the combined strength of the upper and lower column strengths is $2\,CM_y$, if the columns have the same yield strength in all directions. The ratio of total girder strength to total column strength is thus $\sqrt{2}$ times as great as for moments applied parallel to the building axis, making column yield more likely ; and (iii) the direction of the seismic motion relative to the building axis should be considered.

In this paper, a simplified bending and shear model is used to study the inelastic dynamic behavior of a 10-storey frame subjected to bi-directional horizontal ground seismic motion. By changing the column/girder bending strength ratio (α), the column strength required to create a weak-girder frame and the energy input to the structure are

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examined. The effects of bi-directionality and orientation of the input motions on the behavior are also examined.

Seismic Motion

1. Directivity of Bi-directional Seismic Motion

Fig.1 shows the elastic displacement response of a SDOF system subjected to bi-directional horizontal ground motion (El Centro,1940) using T=1.37 secs. and 2.0% damping. The response is clearly bi-directional. The major direction(θ main) is defined as the one in which the maximum displacement occurs, and the minor one is perpendicular to it. They depend on the period and damping of the system, and Fig.2 shows major direction spectra for 2.0%, 10% and 20% damping. An increase in damping (particularly for low levels) clearly reduces the dependence of θ main on T.

2. Input ground motion

For the frame analysis, the bi-directional El Centro(1940) record, scaled by a factor of 1.25, was used. It was resolved into its major and minor directional components based on θ main=31.25°, which was calculated using the fundamental period of the structure (T=1.37secs). Two different directions of attack were then used (major ground motion direction at 0° and 45° to the x-axis of the structure). In addition, the intensity of the minor direction input was scaled by a factor β, which was given different values of 1.0, 0.5, 0.0. This gave a total of six different input motions, which are summarized in Table 1.

Table 1 Summary of Input Ground Motions

			EW Direction	NS Direction
Maximum Acceleration(Amax)			263.3 (gal)	428.2 (gal)
			Major direction	Minor direction
Major Ground Motion	Amax	β=1.0	254.8 (gal)	393.5 (gal)
		β=0.5	254.8 (gal)	196.8 (gal)
		β=0.0	254.8 (gal)	0.0 (gal)
	V_E(T=1.37sec)		152.3(cm/sec)	59.8 (cm/sec)

Major ground motion will be prepared after the El Centro,1940 seismic motion is increased by 1.25-times
$V_E=\sqrt{2\cdot E/m}$: values when major ground motion is inputted into an elastic one mass system (T=1.37sec, h=2.0%)

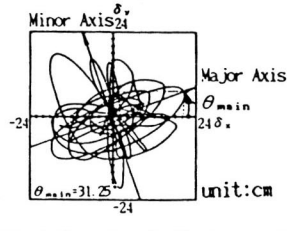

Fig.1 Example of displacement orbit

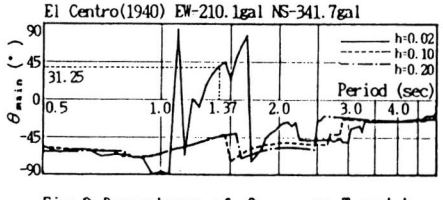

Fig.2 Dependence of θ_{main} on T and h

Structural Model

Fig.3 shows the model of the structure. It is a 10-storey half span frame, containing an infinite number of bays in the X and Y directions, all identical. Each girder-column joint has four degrees of freedom : two horizontal displacement DOF (u and v) and two rotations (θx and θy). As shown in Fig.4a, the girder flexibilities are represented by elasto-plastic rotational springs at each girder end, with the properties shown in Fig.6. The columns are modeled as rigid members (Fig.4a), joined by springs at mid storey height. The force-deflection relationships governing relative rotation and relative horizontal displacement of two contiguous column segments were derived independently. This was done in the interests of simplicity, and avoided the need to formulate a model which includes the interactions among bi-directional shear , bi-directional moment and axial force. The lateral displacements at each mid storey node can be derived from Eq.1. Relative rotations of the column segments are governed by independent elastic springs in each direction.

$$
\begin{aligned}
{}_x\delta_{ij} &= (u_j - \theta_{yj}\cdot h/2) - (u_i + \theta_{yi}\cdot h/2) \\
{}_y\delta_{ij} &= (v_j + \theta_{xj}\cdot h/2) - (v_i - \theta_{xi}\cdot h/2)
\end{aligned}
\qquad \text{-------(1)}
$$

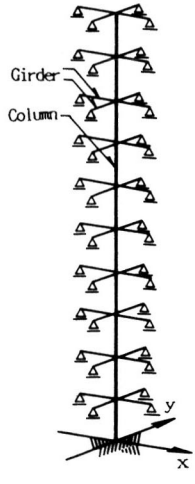

Fig.3 Structural model of building frame

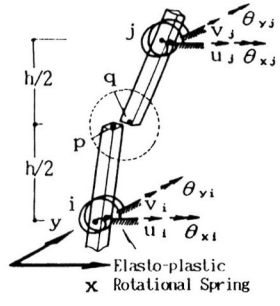

Fig.4a Model of column and girder

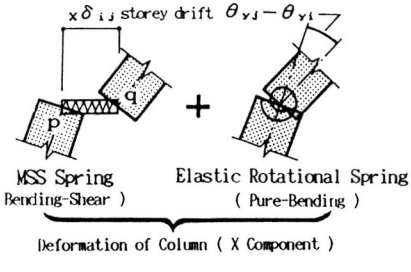

Fig.4b Detail model of column

The storey drift, which includes the displacements caused by both reversed curvature bending and shear, is modeled by a multiple shear spring (MSS) system, shown in Fig. 4b.[7] One is placed at each mid-storey node. The purpose of the MSS model is to simulate a bi-directional inelastic spring, including interaction between the two directions and both kinematic and isotropic strain hardening. It consists of N displacement springs (N>=2) arranged radially around a quadrant of a circle,(Fig. 5a). The two nodes p and q are in the same place, and each spring may have any desired properties. Here, 8 identical springs were used, and each had the force-displacement relationship shown Fig.6. The composite model thus has a yield surface for forces in the x-y plane that is a regular 2N (i.e. 16) sided polygon, which for practical purposes is a circle. The force deflection relationship consists of N/2 piecewise linear segments for forces parallel to any spring and N segments for forces in other direction and is shown in Fig.5c. Equivalent column properties were obtained from the MSS model by ignoring shear flexibility of the column, so

$$\frac{12EI}{h^3} = K_s \qquad \text{------(2a)}$$

$$\frac{2 \cdot {}_cM_y}{h} = Q_y \qquad \text{------(2b)}$$

where Ks and Qy are the elastic stiffness and yield strength of the composite MSS model.

In addition, the spring constant ks, the yield strength qy, of identical shear springs of MSS model can be expressed by Eqs.3a,b using Ks and Qy.

$$k_s = K_s / \sum_{i=1}^{n} \cos^2\theta_i \qquad \text{------(3a)}$$

$$q_y = Q_y / \sum_{i=1}^{n} |\cos\theta_i| \qquad \text{------(3b)}$$

where θi is the angle of the shear spring to the axis X (i.e. θi=(i-1)*22.5°)

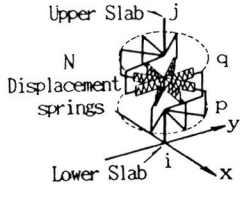

Fig.5a MSS model details (Only 4 springs are displayed)

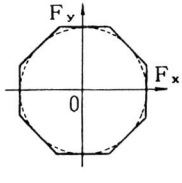

Fig.5b Yield surface (In the case of 4 springs)

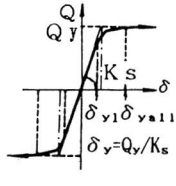

Fig.5c One directional force-displacement response

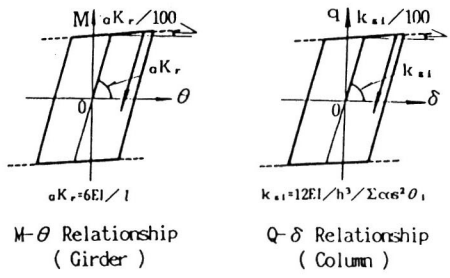

Fig.6 Inelastic member properties

Inelastic Response Analysis

1. Basis

Each bay of the frame was 600 cm long by 360 cm high and had a mass of 13.5 tons. The elastic stiffness of each member was kept constant, but different analyses were conducted for ratios, α, of column to girder yield moment of 1.0, 1.3 and 1.5. These were achieved by keeping the girder properties constant and varying only the yield strength in the MSS model, which was shown in Eq.2b to be directly proportional to the column yield moment. All six inputs shown in Table 2 were applied to each of the three structural models (α=1.0, 1.3 and 1.5).

The inputs are designated by a two character sequence. The first (M=major or D=diagonal) indicates the attack direction and the second indicates the scale factor (0.0, 0.5, 1.0) for the minor direction input. Member properties are given in Table 3. Fig.7 shows the first three mode shapes and periods in the xz and yz planes of the structure. T1=1.37 secs, so θ main =31.25°for h=2.0%.

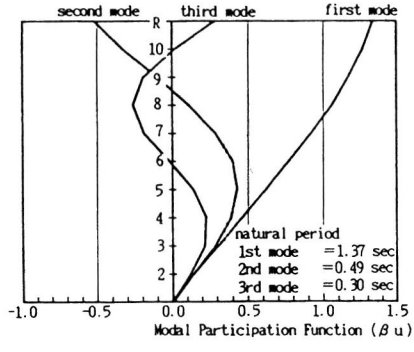

Fig.7 Mode shapes and periods

Table 2 Principal Parameter of Analysis

Analysis Model	Input Direction	Scale Factor for Minor Direction (β)	Column/Girder Strength Ratio (α)
M−10series	major Direction Input	1.0	3 Case α=1.0 α=1.3 α=1.5 for each series
M−05series		0.5	
M−00series		0.0	
D−10series	45° Diagonal Input	1.0	
D−05series		0.5	
D−00series		0.0	

Note: Analysis Model — Input Direction — Scale Factor for Minor Direction(β)
ex.) M−10

Table 3 Member Properties for Analysis

Floor Level	Q_{yi}(ton)	K_i(t/cm)	$I_c/2$ (cm^4) x10^4	I_g (cm^4) x10^4	$_GM_{yi}$(t·cm)
R	8.85	4.92	2.43	4.86	797
10	13.10	7.28	3.59	7.18	1976
9	16.91	9.39	4.64	9.28	2701
8	17.73	9.85	4.86	9.72	3118
7	19.12	10.62	5.24	10.48	3317
6	21.23	11.79	5.82	11.64	3632
5	23.79	13.22	6.53	13.06	4052
4	25.42	14.12	6.97	13.94	4429
3	26.23	14.57	7.19	14.38	4649
2	27.00	15.00	5.23	10.46	4791

Q_{yi} : Storey Yield Shear Force
K_i : Storey-stiffness ($=Q_{yi}/(h/200)$)
I_c : Moment of Inertia of Column $= \frac{200(h+l)h \cdot Q_i}{12E}$, h=360cm l=600cm
$_GM_{yi}$: Bending Strength of Girder($=(_BM_i+_TM_{i-1})/2$, $_BM_i=_TM_i=h \cdot Q_{yi}/2$)

2. *Ductility Results*

Fig.8 shows the distribution of the maximum column ductility factor ($c\mu_{maxi}$) for each α. $c\mu_{maxi}$ is obtained by dividing the maximum MSS model displacement by its yield displacement, dy, taken here as Qy/Ks. Fig.9 shows the distribution of the maximum girder ductility factors ($xG\mu_{maxi}$ and $yG\mu_{maxi}$) for each α.

Comparison of Figs.8 and 9 reveal several trends:

a) As the column to girder strength ratio, α, increases from 1.0 to 1.5, girders ductility demand increases, while that for column decreases. For α=1.0, the girders remain elastic. This is true regardless of attack direction or β.

b) When column ductility demand is large, it is concentrated in a few storeys (Nos 6 and 7). When girder ductility is large, it is uniformly distributed up the building.

c) The largest ductility demands in both columns and girders occur when β=1.0, showing that bi-directional is more damaging than uni-directional input.

d) The largest column ductility demand occurred with the 45° diagonal attack direction regardless of α.

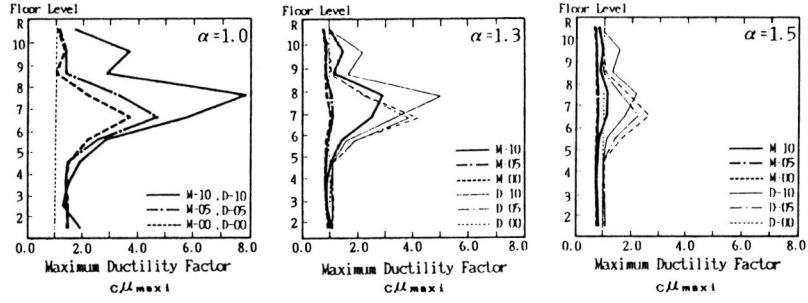

Fig.8 Distribution of maximum column ductility factors

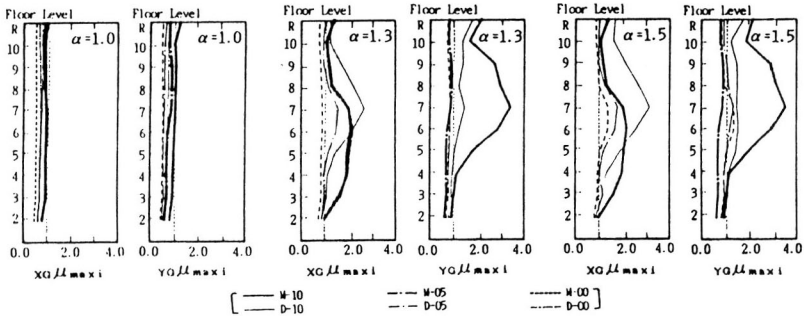

Fig.9 Distribution of maximum girder ductility factors

3. *Displacement and Shear Force Orbits*

Fig. 10 shows the displacement and shear force orbits of the column for the case of α=1.3.

Case I, Major direction input(θ main=0°)

For case of β=0.5, the column shear force orbit remains in the elastic zone. However the maximum displacement slightly exceeds dy because the definition of dy implies sharp yielding, whereas the MSS model provides gradual yielding. When β=1.0, the displacements in both X and Y directions approximately double, because the strength of the column decreases due to the effect of two-directional interaction.

Case II, 45° Diagonal Input(θ main=45°)

At 45° to the axis, the sum of the girder moment strengths is $2\sqrt{2}\,GMy$, whereas the sum of the column strengths is $2\,CMy$. The columns thus become relatively weaker when moments are applied at 45° ,and the column displacements increase significantly for the 45° diagonal attack direction. When β=0.5, both column shear force and displacement are quite strongly directional.

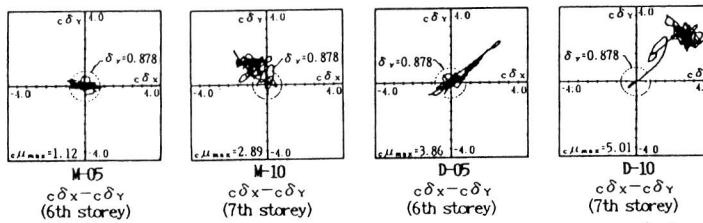

Fig.10a Displacement orbit

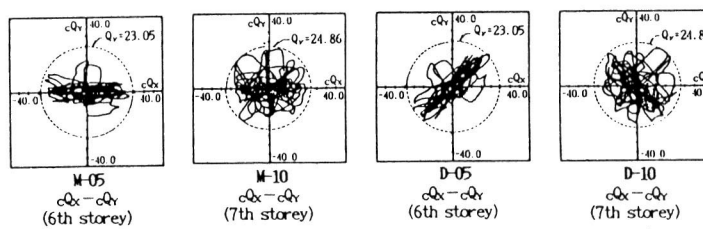

Fig.10b shear force orbit

4. Total Energy Input

Figs. 11 show the time history of absorbed energy in both X and Y directions, for β=1.0 and major direction input. In the figures, the total absorbed energy is divided into damping energy (W d), column hysteresis energy (CW h), girder hysteresis energy (GW h) and kinematic energy (W k). The envelope is the total energy input(E), which is insensitive to α. However, as α changes form 1.0 to 1.5, most of hysteresis energy moves from the columns to the girders.

Fig. 12 shows equivalent velocities (VEX, VEY) obtained from the total energy input in the X and Y directions using Eqs.4a,b.

$$V_{EX} = \sqrt{2 \cdot E_X / \sum_{i=1}^{10} m_i} \qquad \text{------(4a)}$$

$$V_{EY} = \sqrt{2 \cdot E_Y / \sum_{i=1}^{10} m_i} \qquad \text{------(4b)}$$

It shows that:

a) The dependence of VEX and VEY on α is small.

b) With major direction input, VEX is essentially independent of β, but the displacement cdx increases significantly with B,(Fig. 10). This can only occur if the column is relatively weakened by bi-directional effects.

c) For 45° diagonal input, VEX is almost exactly equal to VEY under all circumstances. The motion is thus predominantly at 45°, emphasizing the relative weakness of the column in that direction.

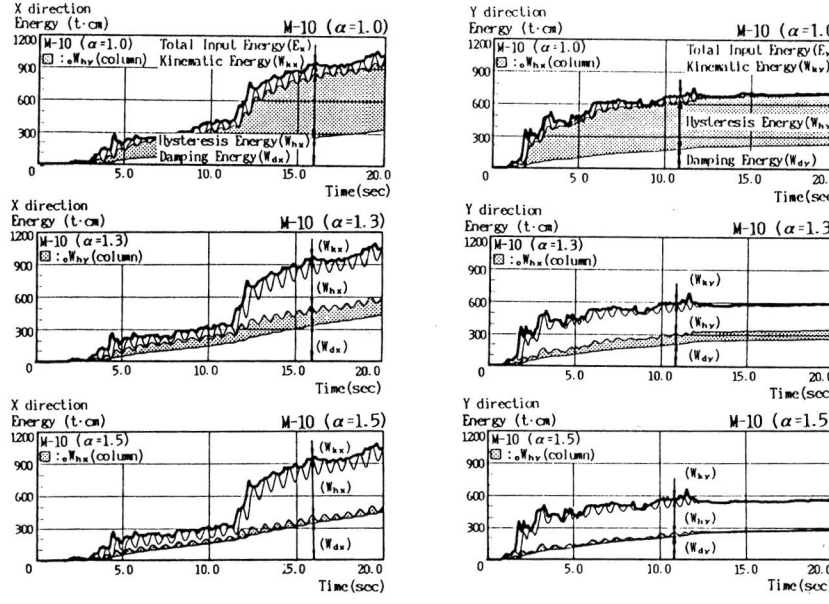

Fig.11 Time history of absorbed energy

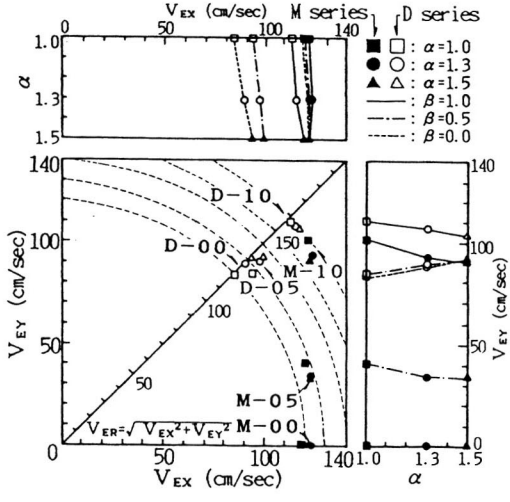

Fig.12 Equivalent velocities

Conclusions

a) To prevent column yielding, the ratio of column moment strength/girder moment strength must have a higher value than that obtained from a uni-directional analysis.

b) The primary causes is that the columns are relatively weakened by the bi-directional structural effects.

c) Loading direction is influential. The most serious condition was found to be when the major direction of the ground motion was applied at 45° to the building axis. Under this input, the structure behaved like a weak-column system even when the columns were 1.5 times as strong as the girders.

Acknowledgement

We appreciate the help of Professor John F. Stanton, Department of Civil engineering of the University of Washington, in making an English version of the paper.

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