Structure Improvement Method Focusing on Lateral Drift

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Abstract

Using the unit load method and the Lagrange multiplier λ, a structure improvement method for minimizing the lateral drift of tall buildings caused by earthquakes or strong winds is proposed in this paper.

Introduction

The problem of drift, horizontal displacement induced by seismic or wind force, is a very serious issue in the design of tall building. Drift is caused by the accumulated deformation of each member, or group of members, and is computed by a black box calculation. Therefore, when we want to control the quantity of displacement by changing the design, it is almost impossible to figure out which member or which member's group should be changed from single computer calculation. This paper aims at proposing a method on drift control, which can find out the most appropriate sizing of members.[1] In recent years, similar drift minimizing techniques have been developed by engineers and researchers in the U.S. and Canada.[2,4,5,6]

Constituents of drift of the top of a building caused by lateral force

I would like to mention the unit load method for calculating displacements derived from the principles of virtual work. Fig. 1 indicates the stress resultants caused by design lateral load on a building . Fig. 2 indicates the stress resultants, of the same building, when unit load is applied to the top of the building. In these figures, k means the sequence number of each member and l_k , w_k mean the length and weight of member k.

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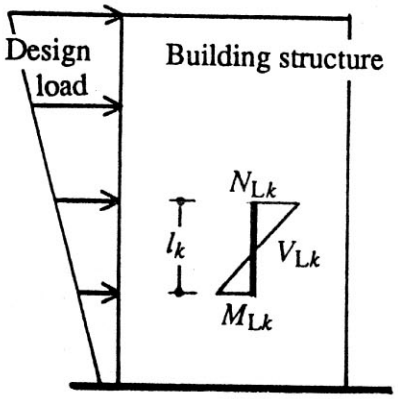

Fig. 1 Stress Resultants Caused by Design Load

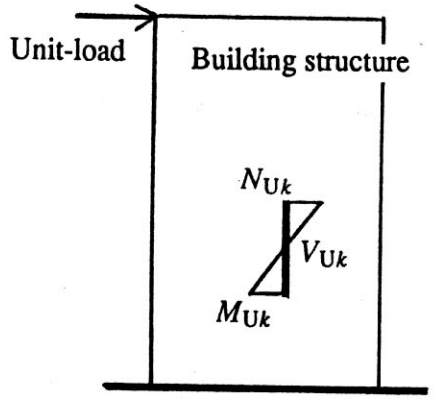

Fig. 2 Stress Resultants Caused by Unit Load

By using the stress resultants of Fig.1, and Fig.2, the lateral displacement of the top of this building can be expressed as

$$D_{top} = \sum_{i=1}^{g} d_i = \sum_{i=1}^{g} (\sum d_k)_i \qquad \text{Eq. 1}$$

where i means the sequence number of each group and g is the number of groups.

$$d_k = \frac{N_{Uk} N_{Lk}}{EA_k} l_k + \int_{l_k} \frac{M_{Uk} M_{Lk}}{E I_k} dx + \frac{\alpha V_{Uk} V_{Lk}}{GA_k} l_k \qquad \text{Eq. 2}$$

The way to minimize the displacement without changing total weight

Total weight of steel can be expressed as

$$W_{total} = \sum_{i=1}^{g} w_i \qquad \text{Eq. 3}$$

where w_i = the weight of group i

Assuming that the cross section of members can be changed only by controlling the thickness of the steel plate without changing its type of figure, the area and moment Inertia and weight of member i, A_i, I_i, w_i, change in proportion to the coefficient α_i which controls the thickness of the plate.

I propose daring assumption for structural deformation as follows.

"The quantity of each member's or group's deformation, which causes the displacement of a certain point of a structure, changes in proportion to the change of its member's or group's stiffness - and this quantity is not affected by the change of any other member's or group's stiffness."

This assumption is exact for statically determinate structures. If there are no great changes in stress distribution, this assumption is good for statically indeterminate structures.[3]

Therefore, the deformation of changed group i can be written as d_i / α_i

It follows that the displacement of the top of the changed building is expressed as

$$D'_{top} = \sum_{i=1}^{g} \frac{d_i}{\alpha_i} \qquad \text{Eq. 4}$$

From the assumption, the total weight of steel is constant. Therefore this is given by

$$W_{total} = W'_{total} = \sum_{i=1}^{g} \alpha_i w_i \qquad \text{Eq. 5}$$

The problem to be solved here is finding the minimum of Eq.4 under the subsidiary condition Eq.5.

By applying the Lagrange multiplier λ , Eq.4 can be rewritten as

$$D'_{top} = \sum_{i=1}^{g} \frac{d_i}{\alpha_i} + \lambda \left(\sum_{i=1}^{g} (\alpha_i w_i) - W_{total} \right) \qquad \text{Eq. 6}$$

By partially differentiating Eq.6 by α_i (i = 1,2,.... g) and λ , and then setting these equations to be equal zero, we can find the values of α_i (i = 1,2,.... g) and λ : which minimize D'_{top}. Consequently we obtain the weight of changed member i as follows

$$w'_i = \alpha_i w_i = \frac{\sqrt{d_i w_i}}{\sum_{j=1}^{g} \sqrt{d_j w_j}} W_{total} \qquad (i = 1, 2, \ldots g) \qquad \text{Eq. 7}$$

where the solutions of this problem α_i (i = 1,2,.... m), λ, can be easily obtained.

In conclusion, the weight of member i, whose section was changed to minimize the displacement of the top, can be obtained by allocating total weight of steel in proportion to the square root of the product of d_i and w_i. This simple formula can be understood intuitively, thus helpful to structural designers.

Application to Tall Building

This method is applied to a statically indeterminate structure, 2-span 15-story high steel structure with 45 columns, 30 girders and 30 cross braces. When a distributed horizontal force is subjected to the structure, the horizontal displacement of the top is figured out to be 39.8 cm. The calculation process is shown in Table 1. The value of di and wi are the results of calculations from the original structure.

The valuable αi is the coefficient indicating the optimum change the cross section of group i members. The term $d'i$ indicates the sum of deformations of all changed members which belong to group i. The summation of $d'i$ (i=1~13) represents the estimated horizontal displacement at the top of the structure (=31.3 cm).

At the top of the changed building, the horizontal displacement caused by the previous force, becomes 29.9 cm. Accordingly, the displacement has been decreased by an estimated value of 1.4 cm : showing this method to be quite efficient. The

original and changed structure have the same total weight (70.4 tons). The displacement at the top can be reduced to be 75 % of original design by this method.

Table 1 Calculation Procedure

group	w_i (tons)	d_i (cm)	$\sqrt{d_i w_i}$	w'_i (tons)	α_i	d'_i (cm)
1	3.16	.42	1.149	1.72	.545	.77
2	3.35	.56	1.371	2.05	.613	.91
3	2.56	.64	1.285	1.93	.751	.86
4	1.95	.76	1.216	1.82	.935	.81
5	6.32	12.90	9.030	13.54	2.141	6.02
6	6.70	6.71	6.706	10.05	1.500	4.47
7	5.13	1.84	3.068	4.60	.897	2.05
8	3.90	.45	1.334	2.00	.512	.89
9	18.99	11.60	14.844	22.25	1.172	9.90
10	9.50	.17	1.265	1.90	.200	.85
11	3.66	2.00	2.705	4.06	1.109	1.80
12	2.90	1.33	1.964	2.94	1.016	1.31
13	2.27	.46	1.020	1.53	.673	.68
total	70.39	39.84	46.957	70.39		31.32

Conclusion

Design is the product of human creativity. Even if Optimum Design Method is completed by computers, it is unlikely that structural engineers will use computer design directly without any consideration. This paper considers mainly structural deformation and proposes a new method to improve structural design.

This paper has developed a method on the assumption that the stiffness of each member is proportional to its weight. In the future, I would like to proceed with the method considering the change of each member's stiffness corresponding to the change of its depth and width. Furthermore, I will try to study deformation analysis which includes plastic deformation caused by large external loads.

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