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ANALYTICAL MODELLING OF BASE ISOLATED STRUCTURES

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Abstract

A base isolated structure can be divided into three components of elastic superstructure, laminated rubber bearings for vertical load support and dampers for seismic energy absorption, and each component design can be proceeded independently. The response of base isolated structures during earthquakes can be calculated easily in comparison to general structures. This paper treats "Base isolated structure subject to two directional seismic motions" and "Rocking vibration of base isolated structure caused by up-down deformations of laminated rubber bearings".

1. Introduction

Some people might have thought even in old days, "A building could be reserved from destruction, if it is not tied to the foundation during earthquakes". The oldest document mentioning base isolated systems was written about 100 years ago. Since then, many ideas have been proposed, but base isolated structure was actually realized by the appearance of laminated rubber bearings.

A laminated rubber bearing has four characteristics; large vertical stiffness, large vertical load capacity, small lateral stiffness less than 1/1000 of vertical stiffness, large elastic deformation capacity. Therefore, a laminated rubber bearing has desirable capability of sustaining building weight safely and supporting it softly in horizontal direction. Additionally, a steel-bar damper, a lead damper and a viscous damper have been developed and these dampers suppress lateral deformation of the base isolation plane and reduce the acceleration raised to the superstructure. They have contributed to realize base isolated buildings.

As its background, we can not miss the progress of experiment technologies, experiment data accumulation and dynamic response analysis using computers. The behavior of base isolated buildings during earthquakes can be figured out using these technologies. Base isolated buildings became admitted as reliable seismic structure not only by specialists but also by ordinary people.

Safety of a general building is to be secured by energy absorption caused by its plastic deformation. Earthquake motion has its own spectrum and elasto-plastic response of a building structure is very complicated. Furthermore, a building structure is statically indeterminate with so many members. If the characteristics of the structure are to be clarified based on the results of experiments on kinematic response of these members, many data and case studies are required. There are so many undefined items that we should consider safety factors at various design stages. Consequently, lots of redundancy are added to the whole structure.

In base isolation design, a superstructure can be elastic and only base isolation devices are subjected to large deformation. Base isolation device consists of dampers and laminated rubber bearings. Dampers are designed to absorb most of earthquake energy. Laminated rubber bearings supporting large vertical load, can be designed to be in elastic condition during big earthquakes. Accordingly, a base isolated structure can be divided into three components; superstructure which

stays elastic, laminated rubber bearings for vertical load support, dampers for seismic energy absorption. Here we see that a base isolation structure is quite simple.

Consequently to the advantage of this structure, each component design can be proceeded independently. Also, laminated rubber and damper can be examined individually by actual size experiments, because the details and boundary conditions of their upper connection to the superstructure and lower connection to the foundation are simple enough.

This simplicity also works effectively for dynamic analysis. The response of base isolated structures during earthquakes can be calculated easily in comparison to general structures. Therefore, rational design can be performed for three components and we can cut down on redundancy which general buildings used to have. To serve this purpose, dynamic analysis are required to be more accurate.

Dynamic response analysis method of base isolated structure is considered here. This paper treats "Base isolated structure subject to two directional seismic motions " and "Rocking vibration of base isolated structure caused by up-down deformations of laminated rubber bearings".

2. Base Isolated Structure Subject to Two Directional Seismic Motion

Seismic design is generally examined on the ground motion of each horizontal direction separately, but there are some problems. When ground motions of two horizontal directions are given simultaneously, their input energy becomes stronger than that of one direction. When certain member becomes plastic by one direction input, this reduces the stiffness and load capacity of another direction. Accordingly, one directional study is not sufficient. To study two directional input is especially required for base isolated structures, because input energy is concentrated on only base isolation plane. This paper presents the analysis method of base isolated structure subject to two directional simultaneous seismic ground motion and provide further consideration with examples.

3. Structural Idealization

3-1 Superstructure

A superstructure is idealized as a multiple mass model with the mass concentrated at each floor, assumed as an elastic body (Fig.1). Each mass is provided with three degrees of freedom; x, y, and θ with the columns and walls of each floor idealized as shear springs (Fig.2). A column has the same spring constant in both x and y direction and a wall has spring stiffness in its plane direction, but none in its width direction.

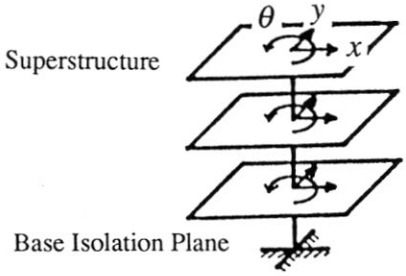

Fig.1 Base Isolated Building Structure

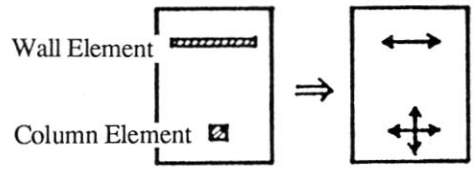

Fig.2 Column and wall

3-2 Isolators

Isolators are modelled as elastic shear springs acting equally in any direction and treated separately from dampers.

3-3 Dampers

Two models are used considering damper yield condition.

a) Two directional spring model (Fig.3)

An elasto-plastic spring is placed in each orthogonal direction and a damper is characterized as bi-linear. Yield condition becomes rectangular without considering correlation between two direction bearing capacities.

b) MSS (Multiple Shear Spring) model (Fig.4)

Many elasto-plastic springs are placed at equianglar position on the same horizontal plane. Thus the correlation between two direction bearing capacities can be considered. The yield condition becomes a polygon shown as Fig.4b. P-δ relationship becomes the curve of Fig.4c when one directional external force is applied. In this paper each damper has 8 elasto-plastic springs. When more springs are used, the polygon becomes closer to a circle, P-δ curve becomes more round. Each spring model is assumed to be bi-linear as two directional spring model.

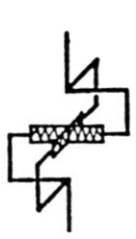

Fig.3a Two Directional Spring Model

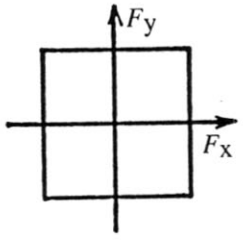

Fig.3b Yield Condition

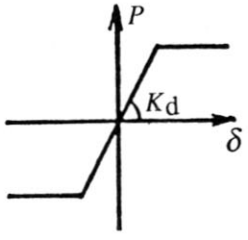

Fig.3c P-δ Relation

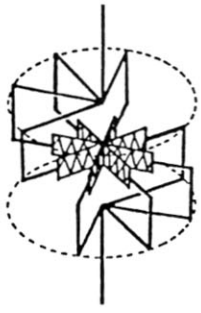

Fig.4a MSS Model

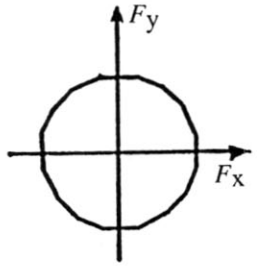

Fig.4b Yield Condition

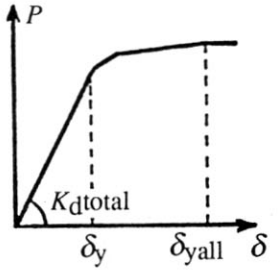

Fig.4c P-δ Relation

When yield condition is a circle, hysteresis characteristic is assumed to be bi-linear from plastic flow theory. When the stress is within the yield circle, its stiffness matrix K becomes (1 in Fig.5):

$$K = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \qquad (1)$$

k : spring constant in one direction

where no two directional interaction exists.

When the stress reaches the yield circle (for example at point 2 in Fig.5), its stiffness matrix K becomes:

$$K = \begin{bmatrix} k\sin^2\theta & -k\sin\theta\cos\theta \\ -k\sin\theta\cos\theta & k\cos^2\theta \end{bmatrix} \qquad (2)$$

where two directional interaction occurs.

Here we concentrate on MSS model. MSS model has 8 elasto-plastic springs and each spring has bi-linear characteristic and spring constant is set to be k_d (Fig.6a). When all elasto-plastic springs yield except the springs orthogonal to load direction, the displacement is set to be δyall.

In MSS model, when the displacement is within the circle of δy, spring constant is $4k_d$. When the displacement crosses the circle, spring stiffness changes as Fig.4c. Furthermore, when the displacement exceeds the circle of δyall, only the springs orthogonal to load direction could stay elastic and all of other springs are changed to be plastic. Then k of equation (2) becomes k_d in MSS model.

When the displacement is changed from point 1 to 5 as described in Fig.6b, the restoring forces at each point in x and y direction are shown in Fig.6c. When the spring constant of every elasto-plastic spring is set to be $k_d = 2.5$, stiffness matrixes of these points are calculated and written in Fig.6d.

At point 3, the same matrix of plastic flow theory is obtained. As one example, the orbits of displacements and restoration forces while MSS model being subjected to earthquake ground motion, are presented by Fig.7a and Fig.7b. Consequently we can conclude that MSS model effectively represents a structural element which has circle yield condition.

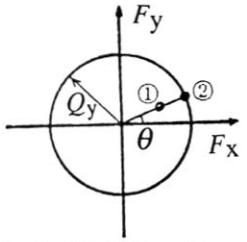

Fig.5 Yield Condition of Flow Theory

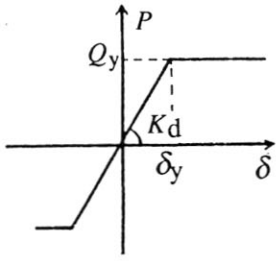

Fig.6a P-δ Relation

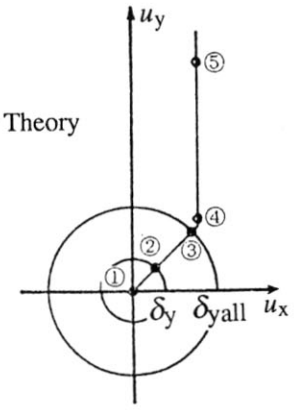

Fig.6b Yield Condition of Displacements

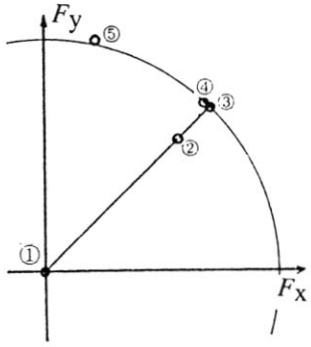

Fig.6c Yield Condition of Shear Forces

① $K = \begin{bmatrix} 1\ 0 & 0 \\ 0 & 1\ 0 \end{bmatrix}$

② $K = \begin{bmatrix} 8.75 & -1.25 \\ -1.25 & 8.75 \end{bmatrix}$

③ $K = \begin{bmatrix} 1.25 & -1.25 \\ -1.25 & 1.25 \end{bmatrix}$

④ $K = \begin{bmatrix} 5.88 & -2.13 \\ -2.13 & 1.62 \end{bmatrix}$

⑤ $K = \begin{bmatrix} 2.5 & 0 \\ 0 & 0 \end{bmatrix}$

Fig.6d Stiffness Changes

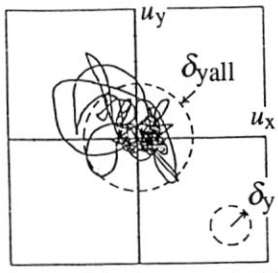

Fig.7a Displacement Orbit

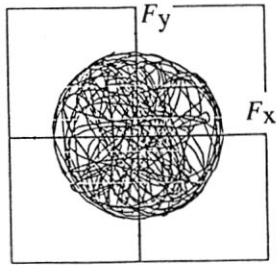

Fig.7b Shear Force Orbit

4. Structures to be Analyzed

The parameters in the example are set for a three storied base isolated building. Analysis model is a four mass model and following two types are studied.

Model 1 : damper as two directional spring model
Model 2 : damper as MSS model

4-1 Superstructure

9 columns are located on each floor as shown in Fig.8 and walls are placed in the center of the building in NS direction. Thus the superstructure is free from eccentricity in stiffness and weight.
The parameters of the superstructure are shown in Table 1.

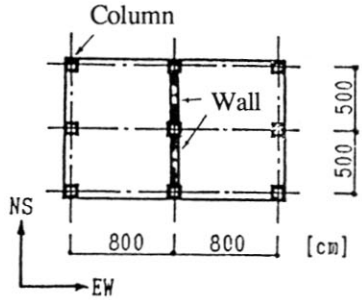

Fig.8 Plan of Building

Table 1 Kinematic Properties of Building

Story	Weight (t)	Rotational Inertia	Stiffness(t/cm)	
RF	100	3.0×10^7	X	Y
3F	150	4.5×10^7	360	720
2F	150	4.5×10^7	450	900
1F	200	6.0×10^7	495	990

4-2 Base Isolation Plane

9 isolators and dampers are used in this example and each one has the same size and characteristics. Therefore we consider the mechanical properties in the total of them.

a) Isolator : An isolator is placed under a column and modelled as an elastic spring. To set the first natural period to be 3.0 sec without dampers, the total of spring constants of 9 isolators is specified to be $k$total = 2.682 tf/cm.

b) Damper : A damper is located at the same place of an isolator. To set the yield shear coefficient to be 0.1, the total of trigger levels of 9 dampers is specified to be $Q_y$total = 60tf and the yield displacement δ_y to be 1.0 cm.

Then the total initial stiffness becomes $k_0$total = 60tf/cm. The resultant of isolators and dampers has the total initial stiffness of 62.682tf/cm and the total trigger level of 62.682tf. These values correspond to those of Model 1. MSS model is idealized by 8 elasto-plastic springs, so individual spring must have 1/4 stiffness and 1/5.027 yield force of Model 1.

4-3 Viscous Damping

On condition that dampers are not attached, 0.5% damping is provided to each mode.

4-4 Input Earthquake Ground Motion

The acceleration records of El Centro earthquake 1940 is used here as input earthquake motion. As standard, the maximum amplitude of records is defined to be 210.1cm/sec^2 in EW direction, and 341.7cm/sec^2 in NS direction. Input levels hereafter are expressed by the multiplication of this standard.

5. Analysis Results

A base isolated structure subject to two directional ground motion is examined using above-mentioned analysis method, seismic inputs and buildings. We compare MSS model with two

directional spring model in 5-1, then study two directional interaction caused by different input scales in 5-2.

5-1 The Comparison between MSS and Two Directional Spring

To examine the influence of two directional interaction, analysis are performed for these two models. As input, we use two directional earthquake ground motion whose multiplication scale is 1.0 in both EW and NS direction Two inter-story displacements are calculated as listed in Table 2:

y_1 : between the foundation and the first floor in NS direction

y_2 : between the first and second floor in NS direction

Table 2 shows that the use of MSS model reduces the inter-story displacement of the superstructure. This reason can be explained as follows, considering Akiyama's "Constant input seismic energy theory". Because of different models in yield condition, more dampers become yielded by using MSS model. Then the inter-story displacement between the base isolation plane and the foundation becomes larger and the base isolation plane absorbs more energy. Consequently input energy to the superstructure is reduced.

Table 2 Maximum Interstory Displacements

Model	y_1(cm)	y_2(cm)
1	6.38	0.064
2	6.74	0.055

5-2 Input Scale and Two Directional Interaction

In this example, the scale of NS direction is 1.0, the scale of EW is specified as 0.5, 1.0, 2.0. Fig.9 a ~ Fig.9 f show the displacement orbits of MSS model and two directional spring model.

Upper three graphs of two directional spring model show that only the displacement of EW direction is increased relating to input scale change. But the response of NS direction is not influenced by the change of EW directional input, because two directional interaction is neglected in this model.

Lower three graphs of MSS model show that the displacements of both directions are influenced, not only in maximum amplitude but also in its shape. It is difficult to simply judge the influence by their amplitudes, because the centers of some vibrations are shifted from the origin of the coordinates. We add the total stroke value of displacement in these graphs. According to these values, we can find the response of NS direction is increased by the change of EW directional input.

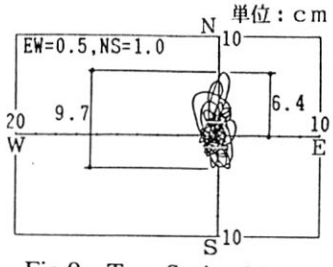

Fig.9a Two-Spring Model

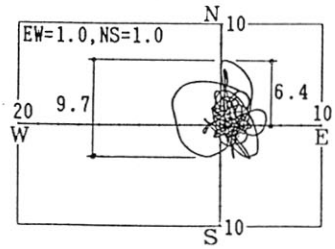

Fig.9b Two-Spring Model

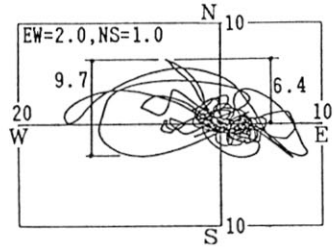

Fig.9c Two-Spring Model

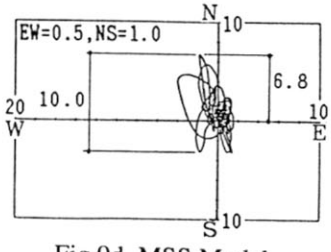

Fig.9d MSS Model

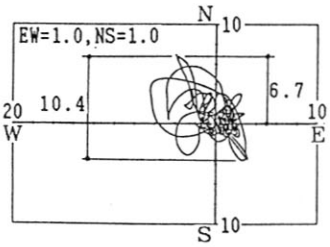

Fig.9e MSS Model

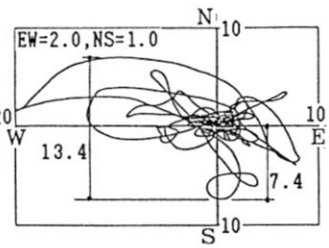

Fig.9f MSS Model

6. Rocking Vibration of Base Isolated Structure

In a laminated rubber bearing, a thin rubber is sandwiched between thin steel plates and compressed by large vertical force, so the rubber becomes liquid-like state. This provides the bearing with capabilities of large vertical stiffness and large flexibility in lateral direction. Thus a laminated rubber bearing has been usually examined in its compressive state with stable mechanical properties. Therefore, a base isolated structure has been designed on the precondition that tensile force is not applied to the bearings.

In the study of a base isolated structure applied to a nuclear power plant, laminated rubber bearings subject to tensile force have been experimented. Fig.10 is derived from the results of these experimentations. From this graph, we find that a laminated rubber bearing can be easily extended in vertical direction by tensile force, because the rubber does not become liquid-like state with tensile force.

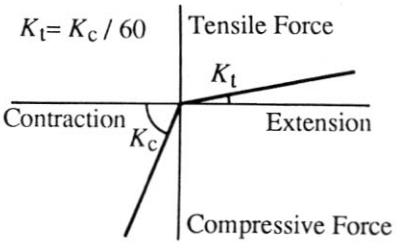

Fig.10

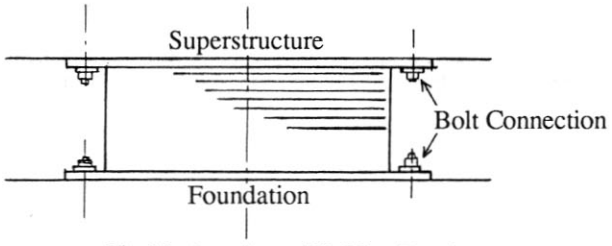

Fig.11 Laminated Rubber Bearing

A laminated rubber bearing is connected to the superstructure and to the foundation by many bolts placed around the rim of circular steel plates attached to the top and bottom of the bearing, as shown in Fig.11. When compressive force is applied to the bearing, the force is not transmitted into these bolts. The weight of the superstructure is uniformly distributed on the bearings and supported by the foundation. On the other hand when the bearings are pulled up, this tensile force is transmitted by these bolts. Then the circular steel plates attached to the top and bottom of the bearing are bent and the bearing is stretched.

As summarized, a laminated rubber bearing subject to tensile force has two characteristics, it is easily stretched and its steel plates are easily bent by tensile force. With these characteristics, the bearing is found to be nonlinear in the relationship between vertical force and deflection, having high stiffness against compressive force and low stiffness against tensile force. Therefore we conclude that large pull-up force is not caused at the bearing although uplift deformation occurs there. We should idealize the bearing as a nonlinear model when tensile force is applied.

In conventional analysis, the bearing is assumed to have a linear characteristic in both compressive and tensile region. The pull-up force caused by uplift deformation is clearly over-estimated and the rubber shall be broken if this calculated force is applied. Thus base isolated structures have been designed not to cause uplift deformation as mentioned above. But large pull-up force is actually not caused at the bearing, so we should pay more attention to uplift deformation. In this paper, we adopt a nonlinear model as a laminated rubber bearing and perform dynamic response analysis, then study the behavior of the base isolated structure, pull-up forces and uplift deformations.

7. Critical Shear Force Coefficient of Uplift Occurrence

Pull-up force to a laminated rubber bearing is caused by over-turning response to lateral earthquake ground motion. When it exceeds the static compressive force of the building weight, uplift phenomenon happens.

A building is assumed to be a rigid body and its height, width, weight are set to be H, B, W respectively. The shear force at a base isolation plane is defined as αW and lateral input force is applied to the point of $H/2$. When uplift deformation occurs on the laminated rubber bearing at edge

of the building, α is calculated as listed in Table 3. In Table 3, n is the number of laminated rubber bearings.

Table 3 Critical Shear Force Coefficients of Uplift Occurrence

n	2	3	4	∞
α	$\frac{B}{H}$	$\frac{2B}{3H}$	$\frac{5B}{9H}$	$\frac{1B}{3H}$

For example, some base isolated buildings in Japan have the shear force coefficient of α = 0.15 ~ 0.20. By setting α = 0.20, the critical height-width relationship with which uplift phenomenon happens is obtained for each n as follows:

$$n = 2, 3, 4, \infty \qquad H/B = 5.0, 3.33, 2.78, 1.67$$

During big earthquakes, shear force coefficient of general buildings is usually 0.5 to 1.0. This means that general buildings have more tendency to tumble down than base isolated buildings.

8. Analytical Model

Rocking dynamic analysis of a base isolated building (Fig.12) are performed, with setting n = 3, H/B = 4.0 which is determined according to Section 7. A superstructure is assumed to be a rigid body and its mass is provided with three degrees of freedom in lateral, vertical and rotational direction. Lateral and vertical earthquake ground motions can be used simultaneously as input and gravity force of the building is always to be applied.

As input, we use an artificial earthquake ground motion as follows: the maximum acceleration is about 400cm/sec^2, the velocity response spectra with 5% damping are 100 cm/sec. The duration time is 60 sec. 4 cases of response analysis are performed here as indicated in Table 4

9. Results of Response Analysis

The maximum value of horizontal displacement is about 30cm and the maximum value of acceleration is 180 cm/sec^2. The horizontal acceleration of the input is set to be about 600 cm/sec^2, so the response acceleration of the superstructure is reduced to 1/3. The maximum of plane shear force is about 250 ton, then the shear force coefficient α becomes 0.18.

Table 4 Analysis Cases

Input Ground Motion	Mechanical Property of Rubber	
	Linear	Nonlinear
Horizontal Motion (x1.5) without Up-down Motion	Case 1.L	Case 1.N
Horizontal Motion (x1.5) with Up-down Motion (x0.5)	Case 2.L	Case 2.N

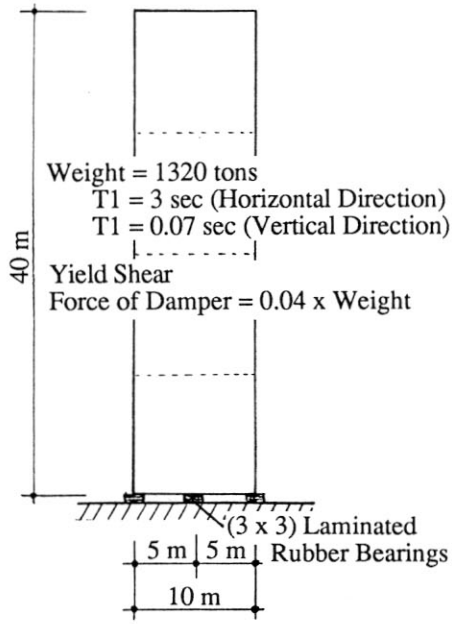

Fig.12 Analysis Model

On the condition of $n = 3$, $H/B = 4$ and $\alpha = 0.18$, uplift phenomenon occurs at the edge bearing according to Section 7. Fig.13 shows the time history of vertical displacement of the bearing at edge of the building. In this figure, the displacement over the line of 0 indicates uplift occurrence.

In Fig.14, the displacements in both horizontal and vertical direction of 3 bearings in Case 2.N are plotted. Fig.14a shows the left side bearing is uplifted when the building is swayed to the right. Fig.14c shows the right side bearing is uplifted when the building is swayed to the left. Fig.14b shows the center bearing moves almost horizontally. But we can find a little vertical raise at the largest horizontal displacement occurrence. When the bearing at edge is uplifted, the center bearing is also slightly displaced upward.

Fig.15 indicates the vertical force and displacement of the left side bearing in Case 1.L and Case 1.N. The point indicated by "1.L" is the pull-out force working on the linear bearing model, and the point indicated by "1.N" is the pull-out force on the nonlinear bearing model. This figure tells us that the uplift deformation is increased by using nonlinear model, but on the other hand the pull-out force is remarkably reduced.

Fig.16 indicates this relationship in Case 2.L and Case 2.N and describes the similar tendency to Case 1.

Fig.17 to Fig.20 show time history response of the vertical displacement of the superstructure's mass. Fig.17 shows the results of Case 1.L. Since the input does not include up-down motion and the bearing is modelled as a linear one, the mass keeps the position settled by the building weight and vibration does not occur. Fig.18 shows the results of Case 1.N. The input also does not include up-down motion but the bearing is modelled as a nonlinear one, so the mass is raised when the bearing at edge is uplifted. Fig.19 shows the results of Case 2.L. The input includes up-down motion, so up-down vibration occurs at the mass. Fig.20 shows the results of Case 2.N. The input includes up-down motion and the bearing is modelled as a nonlinear one, so the largest up-down vibration occurs. The vertical displacement caused by rocking has the similar scale to the vertical response due to up-down ground motion.

Fig.21 to Fig.24 show time history response of the vertical acceleration of the superstructure's mass. These figures are arranged to indicate the same cases as Fig.17 to Fig.20. Fig.21 indicates that the value is always 0. Fig.22 shows the results of Case 1.N using a nonlinear model as a bearing. Right after the mass is raised, vertical vibration whose maximum acceleration is about 90 cm/sec^2 occurs suddenly . Up-down motion and horizontal earthquake ground motion are applied simultaneously in Fig.23 and Fig.24 and the bearing is modelled as a linear one in Fig.23, as a nonlinear one in Fig.24. In both cases, the maximum acceleration becomes 300 cm/sec^2. This value is three times of the vertical acceleration caused by rocking. Accordingly uplift phenomenon caused by rocking is considered to be allowable in base isolation design.

10. Conclusion

New analytical models of base isolated structure are proposed in this paper as follows: MSS model as a damper which has circle yield condition considering two directional interaction, a laminated rubber bearing model which is elastic against horizontal force and nonlinear against vertical force. This paper treats 2 directional vibration problem and rocking problem separately, to figure out each response of a base isolated structure clearly. These problems could be considered simultaneously for accurate understanding of its behavior. The following are some points I have noticed in this study.

1) Base isolated structure is a quite simple structure, where a super structure is assumed to be elastic and a base isolation plane takes charge of energy absorption. Accordingly response analysis could be incorporated into design process, as far as input earthquake ground motion is properly selected.
2) If we use 3 dimensional frame model as a superstructure and put these proposed models under each column, we could produce a quite accurate structural model.

3) As the next step, I would like to perform dynamic analysis considering the influence of vertical force on the horizontal mechanical properties at a laminated rubber bearing.

11. Acknowledgement

I appreciate the helps of Mr. Kazuhiro Yamamoto and Mr. Kenichi Yoshida in numerical calculations, and Mrs. Masako Yoneda in making English version of the paper.

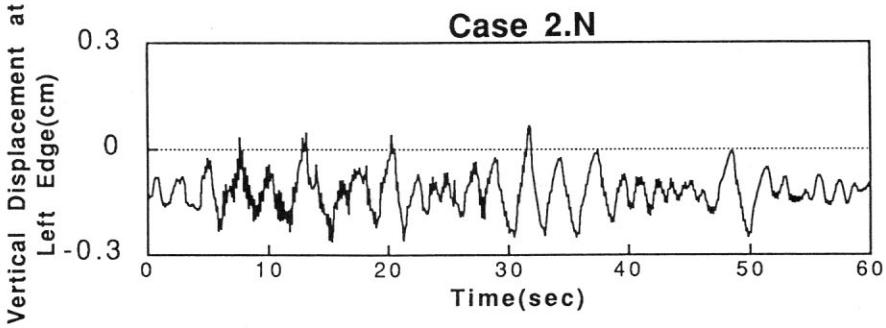

Fig.13

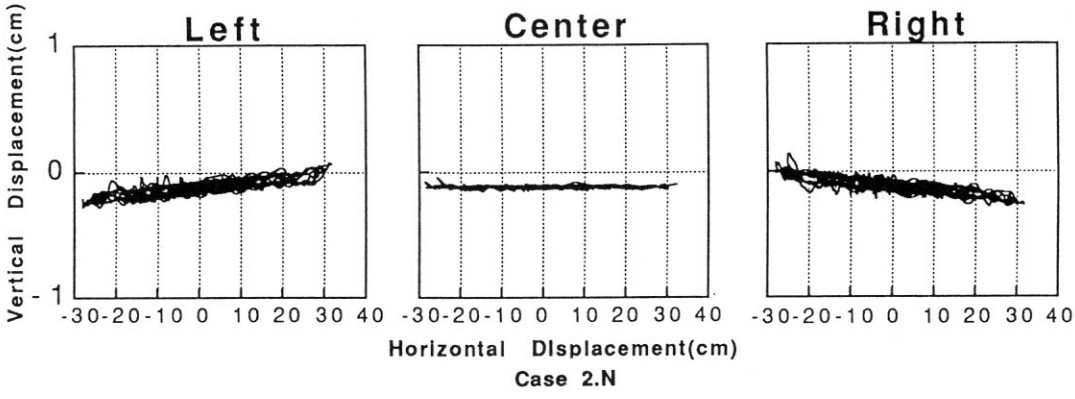

Fig.14

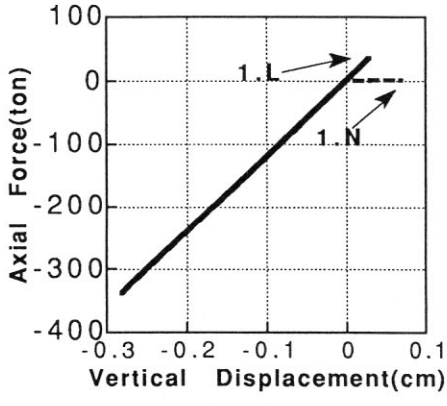

Fig.15

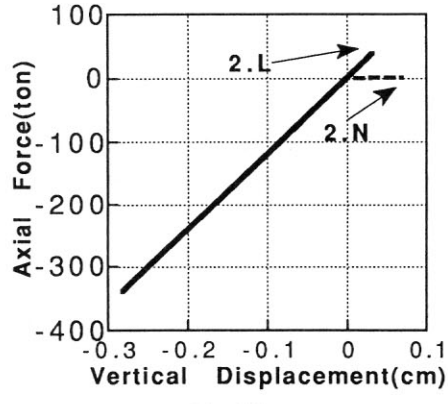

Fig.16

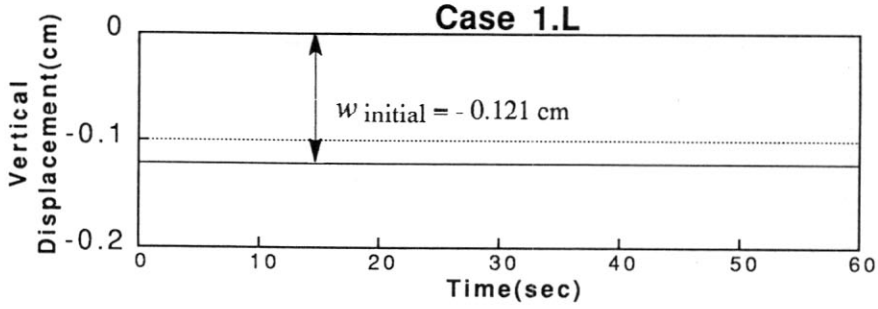

Fig.17

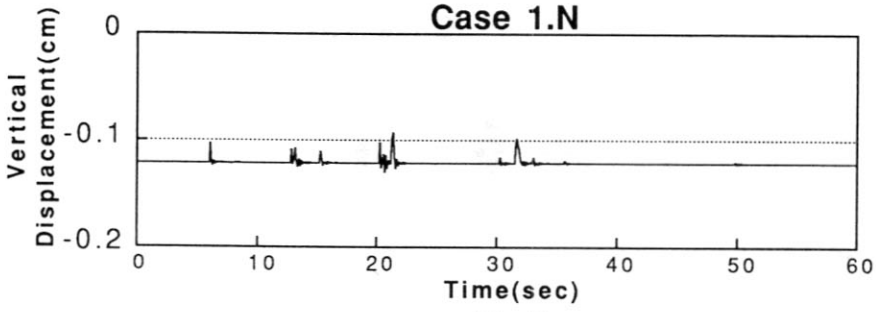

Fig.18

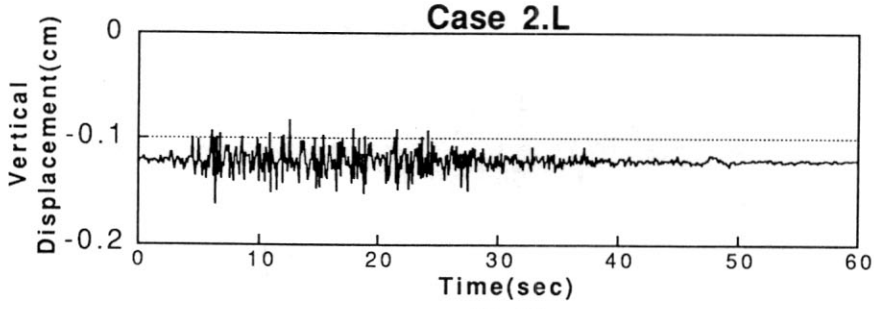

Fig.19

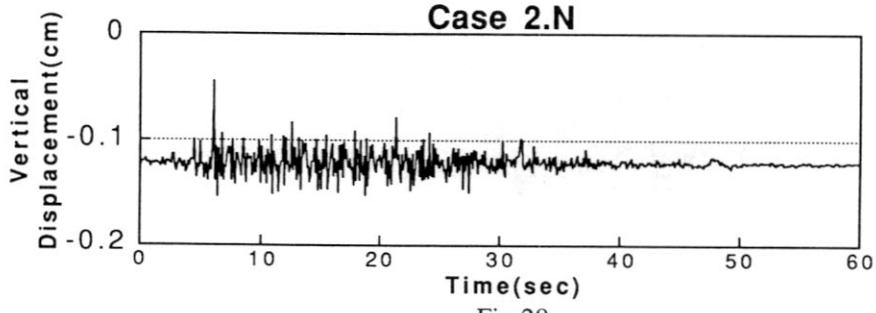

Fig.20

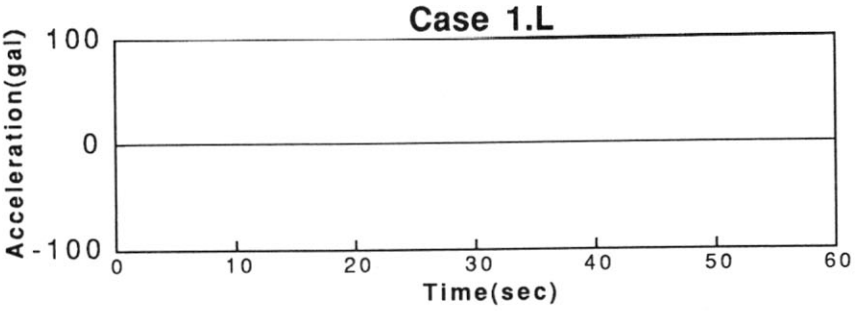

Fig.21

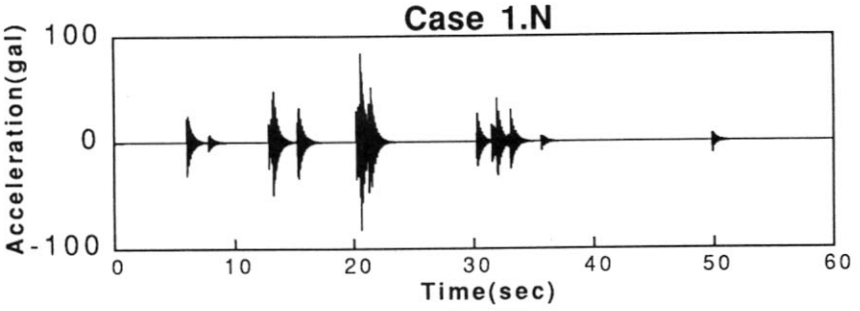

Fig.22

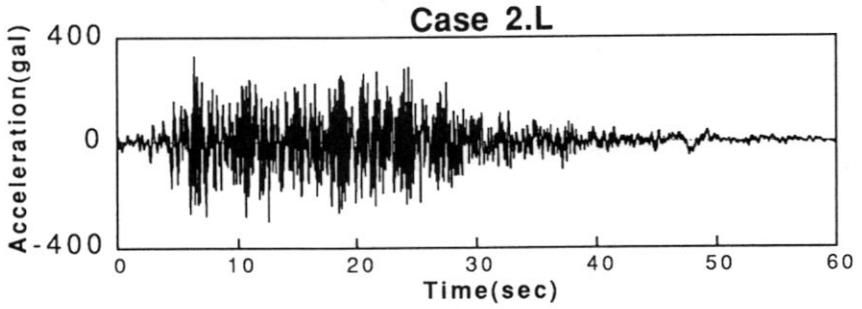

Fig.23

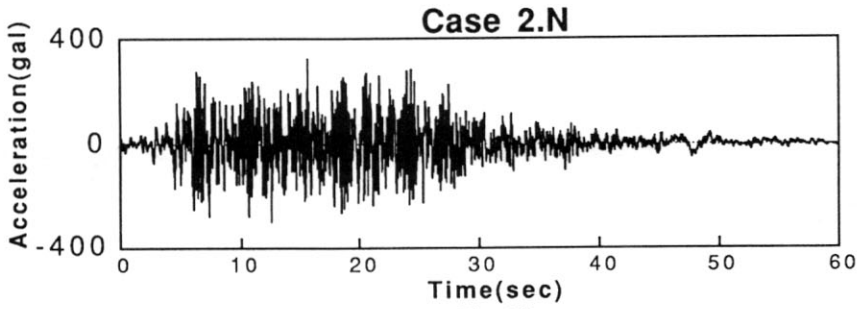

Fig.24